A collaborative vendor-buyer deteriorating inventory system in declining market when trade credit is offered
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ABSTRACT
A collaborative inventory system consisting of single vendor and single buyer is developed to maximize the total profit of the supply chain. The model is developed for prevailing scenario of declining demand due to recession. However, the optimal solution for the supply chain may not be beneficial to both the players. To ensure mutual benefit, a negotiation factor is incorporated to share the profit between the players according to their contributions. To attract the buyer for the joint decision, the vendor offers trade credit for settlement of accounts due against purchases. The units in inventory are subject to constant deterioration and replenishment rate is proportional to demand rate. A numerical example is given to support the proposed model. The sensitivity analysis of model parameter is carried out. It is observed that the percentage of extra total profit is higher when players of the supply chain offer for joint decision.

Keywords: Collaborative Vendor-Buyer supply chain, Supply chain, Trade credit, Deterioration rate, finite replenishment rate, price-sensitive declining demand.

1. Introduction
Due to globalization, the vendor and buyer agree to do business by collaborating in order to survive. The shrinking resources, shortened product life cycle, Customer’s satisfaction, quicker and effective sales and service also forced the players of the business to work jointly. Based on mutual trust, the co-operation includes the sharing of information, resources and profit which are building blocks of an effective supply chain network. The closed cooperation will not only increase the joint profit but also will enable a quicker response to customer demand.

Clark and Scark., 1960 discussed serial multi echelon structured to derive the optimal policy. Banerjee., 1986 developed a joint economic lot-size model for a single vendor, single buyer under the assumption that the replenishment rate of the vendor is finite. Goyel., 1988 extended Banerjee’s model by relaxing the assumption of the lot-for-lot production. Ha and Kim., 1997 analyzed the integrated vendor-buyer inventory system using graphical method. Li et al., 1996 developed a lot-for-lot joint policy when demand is price sensitive.

The promotional scheme of a permissible delay in payments adopted by vendor has received the considerable attention from many researchers. The permissible delay in payments was first explored by Kingsman., 1983. He studied effect of the credit term on the inventory cost.
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Goyal., 1985 and Davis and Gaither derived on economic order quantity under permissible delay in payments. Mandal and Phaujdar., 1989 computed the buyer’s interest earned in the credit term. Shah., 1993 and Jaggi and Aggarwal., 1995 formulated ordering policy for a deteriorating inventory system when delay in payments is offered. Chung., 1998 established analytic results for deteriorating an economic order quantity when trade credit is allowed. For more literature one can refer review articles by Raafat., 1991, Shah and Shah ., 2000 and Goyal and Giri ., 2001. The above stated models were designed either for benefits of the buyer or for the vendor.

Yang and Wee., 2000 discussed a heuristic approach to derive an integrated vendor-buyer inventory model for deteriorating items. Wee and Chang., 2003 used a genetic algorithm to analyze vendor-managed-inventory (VIM) system for deteriorating items. Shah et al., 2008 derived optimal strategy for integrated vendor-buyer deteriorating inventory system by incorporating salvage value to the deteriorated items. Shah and Gor., 2009 studied joint vendor-buyer deteriorating inventory system when units received are uncertain.

Yang and Wee., 2006 developed a collaborative inventory system of a single-buyer and single-vendor for deteriorating items when delay in payments is permissible. They incorporated a negotiation factor to share the profit between the vendor and the buyer according their contributions.

In this paper, a prevailing scenario of recession is targeted when demand decreases with time. An integrated inventory model is developed when units in inventory are subject to constant rate deterioration and the delay in payments is permissible. Due to decrease in the demand of an item, one needs to control its production. The production rate is assumed to be finite and is proportional to the demand rate. A negotiation factor is incorporated to distribute the extra profit sharing between the two players. The sensitivity analysis is carried out to study the relationship between the extra profit and model parameters.

2. Assumption and notations

The mathematical model is derived under the following assumptions and notations:

1. A single – vendor single – buyer supply chain is under consideration.
2. The vendor and buyer share each other’s information.
3. The vendor’s replenishment rate is proportional to the demand rate.
4. Demand rate is decreasing function of time and sales price.
5. Shortages are not allowed.
6. The product in inventory deteriorates at a constant rate \( \theta(0 \leq \theta \leq 1) \). The deteriorated units can neither be repaired nor replaced during the cycle time.
7. Holding costs apply to good units only.
8. Vendor offers permissible delay period to attract the buyer to have joint decision.

We shall discuss three scenarios

1. No vendor - buyer integration and permissible delay in payments.
2. The vendor - buyer integration without permissible delay in payments.
3. The vendor - buyer integration with permissible delay in payments.
   \[ i = 1, 2, 3 \]
The buyer’s related parameters are:
\[ I_{bi}(t) \] : inventory level for scenario i.
\[ A_b \] : Buyer’s ordering cost per order.
\[ C_u \] : Unit purchase cost.
\[ I_b \] : Inventory carrying charge fraction per $ per annum.
\[ L_{mli} \] : Maximum lot-size per delivery for scenario i.
\[ TP_{bi} \] : Annual total profit for scenario i.
\[ ER_{bi} \] : Extra profit sharing for scenario 3 as compared to scenario 1 \( (EP_{bi} = TP_{bi} - TP_{b1}) \).
The vendor’s related parameters are:
\[ p(t, P) \] : annual production rate.
\[ I_{dli}(t) \] : inventory level during the production period for scenario i.
\[ I_{vli}(t) \] : inventory level during the non-production period for scenario i.
\[ A_v \] : set-up cost,$ per cycle.
\[ C_v \] : Unit purchase cost.
\[ I_v \] : Inventory carrying charge fraction per $ per annum.
\[ TP_{vi} \] : Annual total profit for scenario i.
\[ ER_v \] : Extra profit sharing for scenario 3 as compared to scenario 1 \( (EP_v = TP_{vi} - TP_{v1}) \).
The other related parameters for both the vendor and the buyer are
\[ R_i(t,P) = a(1 - bt) - dP, \quad i = 1, 2, 3 \]
Where \( a \) is demand scale parameter
\( b \) is rate of change of demand
\( d \) is price-sensitive demand parameter.
\( P \) is end customer’s retail price
\( a, d > 0 \) and \( 0 \leq b < 1 \)
\[ P_i(t, P) = \lambda R_i(t, P), \quad \lambda > 1; \] annual production rate.
\[ \theta \] : Deterioration rate, \( 0 \leq \theta < 1 \).
\[ TP_i \] : Annual total profit of the system \( (= TP_{bi} + TP_{vi}) \) for scenario i.
\[ \delta \] : Negotiation factor of extra profit sharing between the vendor and the buyer.
\[ M_i \] : Permissible delay period offered by the vendor to the buyer for scenario i.
\[ r \] : Continuous interest rate per annum.
\[ e^{-rM_i} \] : Present value of a unit cost after a time interval \( M_i \) for scenario \( i \).
The decision variables are

- $T_{bi}$: Buyer’s cycle time for scenario $i$.
- $n_i$: Number of shipments from the vendor to the buyer per cycle for scenario $i$.
- $P$: End customer retail price.
- $T_v$: Vendor’s cycle time for scenario $i$.
- $T_{pi}$: Vendor’s production period per cycle for scenario $i$.
- $T_{ni}$: Vendor’s non-production period per cycle for scenario $i$.
- $M_i$: Permissible delay period offered by the vendor to the buyer for scenario $i$.

3. Mathematical model

The levels of inventory for the vendor and the buyer are as shown in figure 1.

![Inventory levels](image)

**Figure 1:** Vendor’s and buyer’s inventory status

The buyer’s inventory level is depleting due to a price-sensitive time dependent demand and a constant deterioration of units in the warehouse. Hence, the buyer’s inventory level can be described by the following differential equation:

$$\frac{dI_{bi}(t)}{dt} + \theta I_{bi}(t) = -R_i(t, P), \quad 0 \leq t \leq T_{bi}, \quad t = 1, 2, 3 \quad (1)$$

The vendor’s total inventory system consists of production and non-production phases. The vendor’s inventory level can be described by the following differential equations:

$$\frac{dI_{vi}(t)}{dt} + \theta I_{vi}(t) = P_i(t, P) - R_i(t, P), \quad 0 \leq t \leq T_{pi} \quad (2)$$

$$\frac{dI_{ni}(t)}{dt} + \theta I_{ni}(t) = -R_i(t, P), \quad 0 \leq t \leq T_{ni} \quad (3)$$
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The initial and boundary conditions are

\[
I_{b1}(T_{b1}) = 0 \tag{4}
\]

\[
I_{a1}(0) = \alpha \tag{5}
\]

\[
I_{b1}(T_{b1}) = \infty \tag{6}
\]

\[
I_{a1}(T_{a1}) = I_{a1}(0) \tag{7}
\]

\[
T_2 = T_{b1} + T_{a1} \tag{8}
\]

and \( T_{b1} = \frac{T_2}{a_i} \tag{9} \)

The solutions of the differential equations (1) – (3) are

\[
I_{b1}(t) = \frac{a^b}{\theta} \left(1 - b^\left(T_{b1} - \frac{t}{\theta}\right)\right) - \frac{b^\theta}{\theta} \left[1 - b^\left(T_{b1} - \frac{t}{\theta}\right)\right] - \frac{\theta}{\theta} \left[1 - b^\left(T_{b1} - \frac{t}{\theta}\right)\right]_0^t \tag{10}
\]

\[
I_{a1}(t) = \frac{a^b}{\theta} \left(1 - b^\left(T_{a1} - \frac{t}{\theta}\right)\right) - \frac{b^\theta}{\theta} \left[1 - b^\left(T_{a1} - \frac{t}{\theta}\right)\right] - \frac{\theta}{\theta} \left[1 - b^\left(T_{a1} - \frac{t}{\theta}\right)\right]_0^t + \alpha \tag{11}
\]

\[
I_{b1}(t) = \frac{a^b}{\theta} \left(1 - b^\left(T_{b1} - \frac{t}{\theta}\right)\right) - \frac{b^\theta}{\theta} \left[1 - b^\left(T_{b1} - \frac{t}{\theta}\right)\right] - \frac{\theta}{\theta} \left[1 - b^\left(T_{b1} - \frac{t}{\theta}\right)\right]_0^t + \alpha \tag{12}
\]

Using \( I_{b1}(0) = I_{mb1} \), the maximum inventory of the buyer is

\[
I_{mb1}(t) = \frac{a^b}{\theta} \left(1 - b^\left(T_{mb1} - \frac{t}{\theta}\right)\right) - \frac{b^\theta}{\theta} \left[1 - b^\left(T_{mb1} - \frac{t}{\theta}\right)\right] - \frac{\theta}{\theta} \left[1 - b^\left(T_{mb1} - \frac{t}{\theta}\right)\right]_0^t \tag{13}
\]

From Appendix A, the values of \( \beta \) and \( \alpha \) are

\[
\beta = \frac{T_1(2anT_1 - abn^2T_1 + a\theta n^2T_1 - ab\theta n^2T_1^2 - 2dPn^3 - dP\theta n^4 T_1)}{2\lambda n^2(a(1 - b^2T_1) - dP)} \tag{14}
\]

and

\[
\alpha = \frac{T_1(2anT_1 - abn^2T_1 + a\theta n^2T_1 - ab\theta n^2T_1^2 - 2dPn^3 - dP\theta n^4 T_1)}{2\lambda n^3} \tag{15}
\]

respectively.

The buyer’s inventory per time unit is

\[
I_{b1} = \frac{1}{T_{bl}} \int_0^{T_{b1}} I_{b1}(t) \, dt \tag{16}
\]

The vendor’s inventory in the integrated two-echelon system is the difference between the vendor’s total inventory and the buyer’s average inventory. Therefore, the vendor’s inventory per time unit is

\[
I_{v1} = \frac{1}{T_{v1}} \left[ \int_0^{T_{v1}} I_{v1}(t) \, dt + \int_0^{T_{v1}} I_{v2}(t) \, dt \right] - I_{bl} \tag{17}
\]

So, the annual total inventory holding cost for the buyer and the vendor are

Buyer’s inventory holding cost;

\[
BHC_b = C_b I_{b1} \tag{18}
\]

and vendor’s inventory holding cost;

\[
BHC_v = C_v I_{v1} \tag{19}
\]
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\[ VHC_i = C_v I_v e^{-\theta} \]  

(19)

respectively.

The annual deterioration costs for the buyer and the vendor are

Buyer’s deterioration cost;

\[ BDC_i = C_B I_B e^{\alpha} \]  

(20)

and vendor’s deterioration cost;

\[ VDC_i = C_v I_v e^{\Theta} \]  

(21)

The annual set-up costs for the buyer and the vendor are

Buyer’s set-up cost;

\[ BSC_i = \frac{A_B}{T_B} \]  

(22)

and vendor’s set-up cost;

\[ VSC_i = \frac{A_v}{T_v} \]  

(23)

The present value of buyer’s unit purchase price is \( e^{-\theta M_1} \) when the vendor offers credit period to settle the account against the purchases. Therefore, the annual purchase costs for the buyer and the vendor are

Buyer’s purchase cost;

\[ BPC_i = \frac{C_p I_p e^{-\Theta M_1}}{T_B} \]  

(24)

and vendor’s purchase cost;

\[ VPC_i = \frac{C_p I_p (T_B + T_1) e^{-\Theta M_1}}{T_v} \]  

(25)

For scenario 1 and 2, \( M_1 \) and \( M_2 \) are zero and for scenario 3, it is \( M_3 > 0 \) because here vendor offers credit period to the buyer.

The annual total profit for buyer and vendor are

\[ TP_{BL} = \frac{P_{MAX}}{T_B} - (BPC_i + EHC_i + BDC_i + BSC_i) \]  

(26)

and

\[ TP_{BL} = \frac{C_p I_p e^{-\Theta M_1}}{T_B} - (VPC_i + VHC_i + VDC_i + VSC_i) \]  

(27)

Using Taylor’s series expansion and (7), we get
\[ T_{1i} \approx \frac{1}{\lambda} \left\{ a \left( 1 + \frac{\lambda - b}{2}T_{2i} - dP \left( 1 + \frac{\lambda T_{2i}}{2} \right) \right) \right\} \]

From (8), we have

\[ T_{1i} \approx \frac{1}{\lambda} \left\{ a \left( \frac{\lambda + \lambda - b}{2}T_{2i} - b \frac{\lambda T_{2i}}{2} + dP \left( 1 + \frac{\lambda T_{2i}}{2} \right) \right) \right\} \]

\[ \frac{\partial TP_{b1}}{\partial T_{b1}} = 0 \]

\[ \frac{\partial TP_{b1}}{\partial P} = 0 \]

3.1 Scenario I

Inventory system without considering vendor-buyer integration and permissible delay in payments:

Here, the buyer makes the decision independently.

So first maximize

\[ TP_{b1} = TP_{b1}(T_{b1}, P) \]

Here \( TP_{b1} \) is a function of two continuous variables \( T_{b1} \) and \( P \), solve

\[ \frac{\partial TP_{b1}}{\partial T_{b1}} = 0 \]

\[ \frac{\partial TP_{b1}}{\partial P} = 0 \]

for \( T_{b1} \) and \( P \), knowing the value of \( T_{b1} \), the vendor’s decision is

\[ \text{maximize } TP_{v1} = TP_{v1}(n) \]

where \( TP_{v1} \) is a function of discrete variable \( n \). The optimal solution of \( n \), say \( n^* \), must satisfy the following condition:

\[ TP_{v1}(n_1^* - 1) \leq TP_{v1}(n_1^*) \leq TP_{v1}(n_1^* + 1) \]

Here, both the vendor and the buyer make their decisions independently. Knowing the buyer’s cycle time and unit sale price and vendor’s number of shipments, total profit

\[ TP_{b1} = TP_{b1} + TP_{v1} \]

can be computed.
3.2 Scenario II

Inventory system considering vendor- buyer integration without permissible delay in payments:

In this scenario, the joint agreeable decision is to take by the vendor and the buyer. So the objective is to maximize

\[
TP_2(P, n_2, T_{22}) = TP_{b2}(P, T_{b2}(T_{22})) + TP_{v2}(P, n_2, T_{12}(T_{22}), T_2(T_{22}), T_{22})
\]  

(35)

Where \(T_{b2}, T_{12}\) and \(T_{22}\) are functions of \(T_{22}\) (from (9), (28) and (29)). Optimize total joint profit with respect to \(P, T_{22}\) and \(n_2\) where \(P\) and \(T_{22}\) are continuous variables and \(n_2\) is a discrete variable.

3.3 Scenario III

Inventory system considering vendor- buyer integration permissible delay in payments:

Under Vendor-buyer integration, the vendor is more beneficial compared to the buyer. So obviously, the buyer will be reluctant to adopt joint decision. To overcome reluctant of the buyer, the vendor may offer some credit terms to attract the buyer for joint decision. A negotiation factor is incorporated to share the extra profit according to their contributions.

The buyer’s extra profit \(EP_b\) is the difference between \(TP_{b3}\) and \(TP_{b1}\). i.e.

\[
EP_b = TP_{b3} - TP_{b1}
\]  

(36)

The Vendor’s extra profit, \(EP_v\), is the difference between \(TP_{v3}\) and \(TP_{v1}\). i.e.

\[
EP_v = TP_{v3} - TP_{v1}
\]  

(37)

Clearly, the total joint profit in scenario 3 is more than the non-integrated total profit in scenario 1. So,

\[
EP_v = \delta(EP_b), \quad \delta \geq 0
\]  

(38)

where \(\delta\) is the negotiation factor.

When \(\delta = 0\), all extra profit is for buyer; when \(\delta = 1\), the extra profit distributed equally among the vendor and the buyer. A large \(\delta\) means that profit is for vendor. So the problem is

\[
TP_3(P, n_2, T_{23}) = TP_{b3}(P, T_{b3}(T_{23})) + TP_{v3}(P, n_2, T_{13}(T_{23}), T_3(T_{23}), T_{23})
\]  

(39)

Subject to

\[
EP_v = \delta(EP_b), \quad \delta \geq 0
\]

where \(T_{b3}, T_{13},\) and \(T_3\) are functions of \(T_{23}\) (from (9), (28) and (29)) and \(n_2\) is a discrete variable.

From (38), obtain \(M_2\) as a function of model parameters. After substituting \(M_2\) into (39), \(TP_3\) is a function of \(n_2, P\) and \(T_{23}\) which are to be computed jointly.
4. Solution procedure

For **scenario 1**, to determine the decision variables, \( P, T_{b1} \) and \( n_2 \), processors is stated in (31), (32) and (34).

For **scenario 2**, to maximize \( TP_5 \) follows steps given below:

Step1: for discrete value of \( n_2 \), to determine optimal values of \( P \) and \( T_{25} \) by equating partial derivatives of \( TP_5 \) with respect to \( P \) and \( T_{25} \) to be zero. Designate the optimal values of \( P \) and \( T_{25} \) for each \( n_2 \) by \( T_{25}(n_2) \) respectively.

Step2: Compute the optimal value of \( n_2 \), (say) \( n_2^* \) which satisfies

\[
TP_5(T_{25}(n_2 - 1), n_2^* - 1, P(n_2^* - 1)) \leq TP_5(T_{25}(n_2), n_2^*, P(n_2^*))
\]

and

\[
TP_5(T_{25}(n_2), n_2^*, P(n_2^*)) \leq TP_5(T_{25}(n_2 + 1), n_2^* + 1, P(n_2^* + 1))
\]

For **scenario 3**, use following procedure to determine the decision variables that maximizes \( TP_5 \).

Step1: From (38), obtain \( M_3 \) in terms of other model parameters including \( n_3 \), \( P \) and \( T_{25} \), substitute it into (39).

Step2: For discrete values of \( n_3 \), set the partial derivative of \( TP_5 \) with respect to \( P \) and \( T_{25} \) equal to zero. Compute the values of \( P \) and \( T_{25} \) for each \( n_3 \) and denote it by \( P(n_3) \) and \( T_{25}(n_3) \) respectively.

Step3: Compute the optimal value of \( n_3 \), (say) \( n_3^* \) which satisfies

\[
TP_5(T_{25}(n_3 - 1), n_3^* - 1, P(n_3^* - 1)) \leq TP_5(T_{25}(n_3), n_3^*, P(n_3^*))
\]

and

\[
TP_5(T_{25}(n_3), n_3^*, P(n_3^*)) \leq TP_5(T_{25}(n_3 + 1), n_3^* + 1, P(n_3^* + 1))
\]

In next section, we illustrate derived model numerically.

5. Numerical example

Consider the following parametric values in proper units:

- Scale parameter, \( a = 30000 \)
- Rate of change of demand parameter, \( b = 2\% \)
- Price sensitive parameter, \( \delta = 35 \)
- Buyer’s purchase cost, \( C_b = $35 \)
- Buyer’s inventory carrying charge fraction per annum per dollar, \( I_a = 20\% \)
- Buyer’s ordering cost per order, \( A_5 = $100 \)
Vendor’s set-up cost, $A_v = $6000

$\lambda = 2.0$

Vendor’s inventory carrying charge fraction per annum per dollar, $L_v = 20\%$

Vendor’s unit cost, $C_v = $20

Negotiation factor, $\delta = 1$

Deterioration rate, $\theta = 10\%$

Continuous interest rate per year, $r = 12\%$

The optimal solution for three scenarios is exhibited in Table 1

Table 1: The optimal solution for various scenarios

<table>
<thead>
<tr>
<th>Scenario i</th>
<th>i = 1</th>
<th>i = 2</th>
<th>i = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>60.64</td>
<td>55.22</td>
<td>55.22</td>
</tr>
<tr>
<td>$d_t$</td>
<td>674</td>
<td>915</td>
<td>915</td>
</tr>
<tr>
<td>$\tau_t$</td>
<td>0</td>
<td>0</td>
<td>0.33</td>
</tr>
<tr>
<td>$n_t$</td>
<td>30</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>$T_{t1}$</td>
<td>0.107</td>
<td>0.105</td>
<td>0.105</td>
</tr>
<tr>
<td>$TP_{b1}$</td>
<td>20636</td>
<td>19104</td>
<td>19104</td>
</tr>
<tr>
<td>$TP_{c1}$</td>
<td>9157</td>
<td>11625</td>
<td>11625</td>
</tr>
<tr>
<td>$TP_{l1}$</td>
<td>29793</td>
<td>30729</td>
<td>30729</td>
</tr>
<tr>
<td>$PETP_{l1}$</td>
<td>0</td>
<td>3.14</td>
<td>3.14</td>
</tr>
</tbody>
</table>

Table 2: Sensitive analysis for the demand scale parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>$P$</th>
<th>$d_2$</th>
<th>$n_3$</th>
<th>$\tau_3$</th>
<th>$Tb_3$</th>
<th>$TP_3$</th>
<th>$PETP_3$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>-10%</td>
<td>51.24</td>
<td>750</td>
<td>25</td>
<td>0.354</td>
<td>0.114</td>
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<td>22136</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>59.28</td>
<td>1077</td>
<td>23</td>
<td>0.319</td>
<td>0.099</td>
<td>39630</td>
<td>40659</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>63.40</td>
<td>1230</td>
<td>23</td>
<td>0.298</td>
<td>0.092</td>
<td>50814</td>
<td>51913</td>
</tr>
</tbody>
</table>

Table 3: Sensitive analysis for the demand rate parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>$P$</th>
<th>$d_2$</th>
<th>$n_3$</th>
<th>$\tau_3$</th>
<th>$Tb_3$</th>
<th>$TP_3$</th>
<th>$PETP_3$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>-20%</td>
<td>55.05</td>
<td>964</td>
<td>21</td>
<td>0.369</td>
<td>0.109</td>
<td>29336</td>
<td>30411</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>55.12</td>
<td>944</td>
<td>22</td>
<td>0.355</td>
<td>0.108</td>
<td>29569</td>
<td>30565</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>55.33</td>
<td>822</td>
<td>26</td>
<td>0.323</td>
<td>0.103</td>
<td>30035</td>
<td>30904</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>55.47</td>
<td>848</td>
<td>28</td>
<td>0.300</td>
<td>0.102</td>
<td>30316</td>
<td>31094</td>
</tr>
</tbody>
</table>
Table 4: Sensitive analysis for the price-sensitive demand parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>$P$</th>
<th>$d_s$</th>
<th>$n_s$</th>
<th>$r_b$</th>
<th>$T_b$</th>
<th>$T_P$</th>
<th>$T_P'$</th>
<th>$PETP_3(%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>-20%</td>
<td>65.72</td>
<td>1023</td>
<td>23</td>
<td>0.268</td>
<td>0.100</td>
<td>44838</td>
<td>45666</td>
<td>1.85</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>59.86</td>
<td>973</td>
<td>23</td>
<td>0.304</td>
<td>0.104</td>
<td>36412</td>
<td>37304</td>
<td>2.45</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>51.43</td>
<td>858</td>
<td>24</td>
<td>1.790</td>
<td>0.109</td>
<td>24504</td>
<td>25457</td>
<td>3.89</td>
</tr>
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<td></td>
<td>20%</td>
<td>48.33</td>
<td>799</td>
<td>25</td>
<td>2.495</td>
<td>0.112</td>
<td>20246</td>
<td>21167</td>
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</table>

Table 5: Sensitive analysis for the vendor set-up cost

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>$P$</th>
<th>$d_s$</th>
<th>$n_s$</th>
<th>$r_b$</th>
<th>$T_b$</th>
<th>$T_P$</th>
<th>$T_P'$</th>
<th>$PETP_3(%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_v$</td>
<td>-20%</td>
<td>54.98</td>
<td>937</td>
<td>22</td>
<td>-0.10</td>
<td>0.102</td>
<td>30858</td>
<td>31235</td>
<td>1.22</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>55.07</td>
<td>933</td>
<td>22</td>
<td>0.155</td>
<td>0.107</td>
<td>29986</td>
<td>30974</td>
<td>3.29</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>55.33</td>
<td>900</td>
<td>25</td>
<td>0.324</td>
<td>0.106</td>
<td>29611</td>
<td>30497</td>
<td>2.99</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>55.45</td>
<td>888</td>
<td>26</td>
<td>0.311</td>
<td>0.107</td>
<td>29440</td>
<td>30278</td>
<td>2.85</td>
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</table>

Table 6: Sensitive analysis for the buyer’s ordering cost

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>$P$</th>
<th>$d_s$</th>
<th>$n_s$</th>
<th>$r_b$</th>
<th>$T_b$</th>
<th>$T_P$</th>
<th>$T_P'$</th>
<th>$PETP_3(%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_b$</td>
<td>-20%</td>
<td>55.20</td>
<td>913</td>
<td>27</td>
<td>0.337</td>
<td>0.093</td>
<td>29996</td>
<td>30928</td>
<td>3.11</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>55.20</td>
<td>914</td>
<td>25</td>
<td>0.339</td>
<td>0.100</td>
<td>29892</td>
<td>30826</td>
<td>3.12</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>55.23</td>
<td>917</td>
<td>23</td>
<td>0.361</td>
<td>0.110</td>
<td>29700</td>
<td>30637</td>
<td>3.15</td>
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<tr>
<td></td>
<td>20%</td>
<td>55.24</td>
<td>914</td>
<td>22</td>
<td>0.340</td>
<td>0.115</td>
<td>29611</td>
<td>30549</td>
<td>3.17</td>
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</table>

Table 7: Sensitive analysis for the buyer’s holding cost

<table>
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<tr>
<th>Parameter</th>
<th>% changes</th>
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<th>$d_s$</th>
<th>$n_s$</th>
<th>$r_b$</th>
<th>$T_b$</th>
<th>$T_P$</th>
<th>$T_P'$</th>
<th>$PETP_3(%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_b$</td>
<td>-20%</td>
<td>55.19</td>
<td>920</td>
<td>23</td>
<td>0.342</td>
<td>0.109</td>
<td>29820</td>
<td>30769</td>
<td>3.18</td>
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<tr>
<td></td>
<td>-10%</td>
<td>55.21</td>
<td>912</td>
<td>24</td>
<td>0.338</td>
<td>0.105</td>
<td>29813</td>
<td>30748</td>
<td>3.14</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>55.22</td>
<td>918</td>
<td>24</td>
<td>0.339</td>
<td>0.104</td>
<td>29775</td>
<td>30709</td>
<td>3.14</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>55.22</td>
<td>913</td>
<td>24</td>
<td>0.340</td>
<td>0.104</td>
<td>29756</td>
<td>30690</td>
<td>3.14</td>
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</table>

Table 8: Sensitive analysis for the vendor’s holding cost

<table>
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<th>Parameter</th>
<th>% changes</th>
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<th>$d_s$</th>
<th>$n_s$</th>
<th>$r_b$</th>
<th>$T_b$</th>
<th>$T_P$</th>
<th>$T_P'$</th>
<th>$PETP_3(%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_v$</td>
<td>-20%</td>
<td>55.21</td>
<td>904</td>
<td>25</td>
<td>0.339</td>
<td>0.106</td>
<td>30071</td>
<td>30956</td>
<td>3.08</td>
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<td></td>
<td>-10%</td>
<td>55.20</td>
<td>916</td>
<td>24</td>
<td>0.340</td>
<td>0.107</td>
<td>29656</td>
<td>30861</td>
<td>4.06</td>
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<tr>
<td></td>
<td>10%</td>
<td>55.21</td>
<td>923</td>
<td>23</td>
<td>0.340</td>
<td>0.106</td>
<td>29389</td>
<td>30601</td>
<td>4.12</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>55.23</td>
<td>925</td>
<td>23</td>
<td>0.336</td>
<td>0.104</td>
<td>29263</td>
<td>30475</td>
<td>4.14</td>
</tr>
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</table>

Table 9: Sensitive analysis for the deterioration rate

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
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<th>$d_s$</th>
<th>$n_s$</th>
<th>$r_b$</th>
<th>$T_b$</th>
<th>$T_P$</th>
<th>$T_P'$</th>
<th>$PETP_3(%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>-20%</td>
<td>55.17</td>
<td>902</td>
<td>25</td>
<td>0.367</td>
<td>0.109</td>
<td>30226</td>
<td>31109</td>
<td>3.05</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>55.06</td>
<td>919</td>
<td>24</td>
<td>0.349</td>
<td>0.108</td>
<td>30004</td>
<td>30934</td>
<td>3.09</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>55.23</td>
<td>924</td>
<td>23</td>
<td>0.340</td>
<td>0.105</td>
<td>29591</td>
<td>30782</td>
<td>4.02</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>55.28</td>
<td>921</td>
<td>24</td>
<td>0.336</td>
<td>0.099</td>
<td>29396</td>
<td>30335</td>
<td>3.19</td>
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</tbody>
</table>

Table 10: Sensitive analysis for the negotiation factor
For scenario 1, the optimal sale price is $60.64 and buyer’s cycle time 0.107 resulting annual demand of 674 units. The buyer’s total profit is $20637. The optimal number of shipments from the vendor to the buyer per cycle is 30. Resulting the vendor’s annual total profit to be $9157 hence the total profit of the vendor and the buyer is $29793.

In scenario 2, the vendor and the buyer make joint decision without allowable credit period. The optimal sale price is $55.22 and the annual demand is 915 units with buyer’s cycle time of 0.105 years. The buyer’s, the vendor’s and total joint profits are $19104, $11625 and $30729 respectively. Here, the total joint profit increases by $936 with respect to scenario1. Since the vendor gains by $2468 and the buyer’s loose by $1532, the buyer’s will be reluctant to adopt joint decision. To attract the buyer, the vendor may offer some credit period. Agreeing to equal sharing (\( d = 1 \)), the optimal credit period for the buyer is 0.33 years. The extra profit of $936 is to be distributed equally between the vendor and the buyer. When this negotiation factor is incorporated, the percentage of extra profit is 3.14%. The concavity of the total profit is as shown in fig.2 for scenario 3.
From Table 2, it is observed that when the demand scale parameter increases, the percentage of total extra profit decreases. It is seen in Table 4 that increase in price sensitive demand parameter increase the percentage of total extra profit seen in figure 3.

It is advised to the players of the supply chain to take the joint decision. From Table 5, it is seen that change in rate of demand parameter; b decreases delay period and the percentage of total extra profit significantly. In Tables 5 and 6, the vendor’s and the buyer’s ordering cost varied from -20% to 20%. Increase in vendor’s ordering cost reduces the percentage of total extra profit. Figure 4 shows that the value of the total extra profit is highly sensitive to the vendor’s set-up cost as compared to the buyer’s ordering cost. Hence it is advantageous to adopt integrated strategy when the vendor’s set-up cost and/or the buyer’s ordering cost increases.

From Table 7, 8 and Table 9, it is seen that the increase in the vendor’s inventory caring charge fraction; I_v, the buyer’s inventory caring charge fraction; I_b and the deterioration rate, increase percentage of total extra profit is almost linear. See figure 5.

The effect of negotiation factor is studied in table 10. It is seen that the total extra profit remains constant because, the negotiation factor relates two players extra profit sharing which does not affect the total profit of the supply chain.

5.1 Conclusion

A collaborative optimal policy is developed for the two players of the supply chain. The demand is assumed to be decreasing function of time and selling price. The production rate is promotional to the demand rate. To encourage the buyer for a joint decision, the vendor offers a credit period. The numerical example suggests that the vendor-buyer collaboration results in an extra profit of $ 936. From the sensitivity analysis it is seen that the rate of change of demand parameter and the deterioration rate are critical parameters. The increase in price sensitive parameter and decrease in demand scale parameter results significant increase in profit when the players of the supply chain take decision jointly. This study helps in constructing an efficient supply chain in present market as demand is decreasing.
Appendix A: Computations in $\alpha$ and $\beta$ in fig.1:

During $[0, \beta]$, the vendor’s inventory level at any instant of time can be described by the differential equation

$$\frac{dI_v(t)}{dt} + \theta I_v(t) = P(t, P), \quad 0 \leq t \leq \beta \quad (A.1)$$

Using $I_v(\beta) = 0$, the solution of differential equation (A.1) is

$$I_v(t) = \frac{\lambda ab}{\theta^2} (1 - e^{(t-\beta)}) - \frac{\lambda e^{(t-\beta)}}{\theta} [a(1 - b\beta) - dP] + \frac{\lambda e^{\beta}}{\theta} [a(1 - b\beta) - dP] \quad 0 \leq t \leq \beta \quad (A.2)$$

Clearly, the production quantity in $[0, \beta]$ is equal to the buyer’s lot-size. So, using log series expansion

$$\beta = T_i \frac{(2an^3 - abn^2 T_i + a\theta n^2 T_i - ab\theta n T_i^2 - 2dPn^3 - dP\theta n^2 T_i)}{2\lambda n^2 (a(1 - bT_i) - dP)} \quad (A.3)$$

and, hence

$$\alpha = T_i \frac{(2an^3 - abn^2 T_i + a\theta n^2 T_i - ab\theta n T_i^2 - 2dPn^3 - dP\theta n^2 T_i)}{2\lambda n^2} \quad (A.4)$$

6. References


A collaborative vendor-buyer deteriorating inventory system in declining market when trade credit is offered
Nita H. Shah, Nidhi Raykundaliya


