ABSTRACT

For preliminary design of tall frames, information regarding stress resultants due to lateral loads is required even before member dimensions are known. For this purpose, the portal and cantilever methods are sought in practice for the specific reason that they do not require cross sectional dimensions for the analysis. The conceptual power of these two methods is such that the solution is accepted without demur by the practicing engineers. This paper delineates yet another procedure which throws more light on the evolution process for determining the shear force in the columns of the storey under consideration. The modified analysis propounded gives almost the same answer as that of the improved portal method. The reasoned supposition put forward for the determination of shear is lucid, efficacious, powerful and realistic. The method does not entail much computational effort and redounds to arriving at acceptable terminal moments in beams and columns of the plane frame.

Keywords: Analysis, approximate methods, area, cantilever method, exact method, improved portal method, lateral load, portal method, short frame, tall frame.

1. Introduction

Tall building design is an iterative process. The design is primarily governed by the lateral loads, viz., wind, earthquake and blast loads. For designing the columns, beams and beam-columns, to begin with knowledge regarding the stress resultants caused by these load is imperative even before the cross sectional dimensions are known (Selvam 1996). Approximate solutions for multistory buildings subjected to lateral loads are accepted in lieu of exact analysis (Schultz 1992, Selvam 2010). Therefore, for preliminary design, the two approximate methods, namely, the portal and the cantilever methods are resorted to for the unique reason that they do not call for cross sectional properties for their operation. The potency of the methods is such that many analysts accept their prediction without objection, and use the results in the beginning stages. These two methods have been successfully exploited in the construction of tall buildings all over the world. For short buildings, the portal method and tall buildings, the cantilever method is adopted. These approximate methods are based on some assumptions (Wilbur 1935, Mortelmans et al. 1981,Selvam and Bindhu 2010). The features of short building and tall building are delineated in reference 6 (Selvam and Bindhu 2011). The portal method has two versions which will be reviewed below succinctly.

2. Review of the portal method
There are two versions of the portal method. One is the simplified portal method which is described in Norris et al. (1976). The other is the improved portal method which is dealt with in Wang (1983). Both these two and the method to be proposed subsequently in the ensuing pages assume “Hinges are located in the middle of all the columns and the beams”.

In the simplified portal method, the storey shear is distributed among the columns considering from the left side as

\[ 1 : 2 : 2 : \ldots \ldots : 1 \]  

(1)

The consequence of this assumption is

(a) Beam terminal moments in any floor are same in all the bays.

(b) The shear in all the interior columns is same. Again, the magnitude of the shear in the two exterior columns is same.

These are the two limitations which are put right in the improved portal method as follows:

Let \( l_1, l_2, l_3, \ldots, l_n \) be the spans of the various bays in any storey. Then the storey shear is distributed among the columns in proportion to the tributary length of the columns (pro rata) as

\[ l_j : (l_1 + l_2) : (l_2 + l_3) : \ldots : l_n \]  

(2)

Assumption No.1 or No. 2 along with the hinge assumption renders the frame statically determinate. Then using the equilibrium conditions, all the beam and column end moments are found. It is evident that Eq. (2) is an improvement over Eq. (1).

3. Aim of the paper

The objective of the paper is to advance a procedure for the determination of the shear in the columns. The procedure is easy to comprehend, elegant and is amenable to reasoning. It is found that the results of the proposed modification are very close to that of the improved portal method.

4. Governing parameters in slender and short structures

In many theories of structural analysis (Timoshenko and Young1963), the governing parameters happen to be the area of cross section, \( A \) and the moment of inertia, \( I \). The following are a few examples.

(a) Slender beams : longitudinal fibre stress, \( f = \frac{My}{I} \)

(b) Short beams : Shear force, \( F_s = f_s A \)

(c) Slender Columns : Euler critical load, \( P_e = \frac{\pi^2 EI}{l_e^2} \)

(d) Short Columns : Crushing load, \( P_c = f_c A \)
Where \( y \) = distance of the fibre from the neutral axis

\[ f_s = \text{average shear stress} \]
\[ E = \text{Young's modulus} \]
\[ l_e = \text{effective length} \]
\[ f_c = \text{crushing stress} \]

From the above examples, it is evident that in short systems, the governing parameter is the area of cross section, \( A \).

5. Evolution of the proposed theory

A two storey short rectangular frame with reticulated elements with three bays is shown in Fig.1. Let \( l_1, l_2 \) and \( l_3 \) be the span lengths and \( A_1, A_2, A_3 \) and \( A_4 \) be the areas of the columns counted from left end.

![Figure 1: Plane Moment Resistance Frame](image)

The height of each storey is ‘\( h \)’. Let \( R \) and \( S \) be the nodal loads acting at the top and bottom storeys as indicated in Fig.1. Similar to the portal and cantilever methods, here also a hinge is assumed in the middle of all the members. The frame is split into three single bay frames carrying nodal loads \( R_1, R_2 \) and \( R_3 \) in the top storey and \( S_1, S_2 \) and \( S_3 \) in the bottom storey. The splitting line bisects the area of each interior column into two halves. That is the
area of the interior columns in the split frames will be $0.5A_2, 0.5A_3, \text{etc.}$. Let $Q_1, Q_2$ and $Q_3$ be the combined area of each split frame. That is

$$Q_1 = A_1 + 0.5A_2$$

$$Q_2 = 0.5A_2 + 0.5A_3$$

$$Q_3 = 0.5A_3 + A_4$$

Now

$$Q_1 + Q_2 + Q_3 = A_1 + A_2 + A_3 + A_4 = A = Q$$

In Eq. (3), (4), (5) and (6), $A_1, A_2, A_3$ and $A_4$ are unknowns. To determine the same, the following hypothesis is set forth.

"Area of each column in the original frame is directly proportional to its tributary length."

That is,

$$A_i \propto L_i$$

Where $L_i$ is tributary length of $i^{th}$ column. Considering the constant of proportionality as unity, the hypothesis leads to

$$A_1 = 0.5l_1$$

$$A_2 = 0.5l_1 + 0.5l_2$$

$$A_3 = 0.5l_2 + 0.5l_3$$

$$A_4 = 0.5l_3$$

Check $A_1 + A_2 + A_3 + A_4 = A = l_1 + l_2 + l_3$

5.1 To determine the nodal loads $R_1$, $R_2$ and $R_3$ in the split frame.

The nodal load acting in the left most split frame in the top storey is $R_1$. Therefore, the shear in the top most storey becomes $R_1$. The following postulation is offered for consideration.

"The storey shear is resisted by the combined areas of the two columns in each storey of the split frame".

This means

$$R_1 = q, Q_1$$

$$R_2 = q, Q_2$$

$$R_3 = q, Q_3$$

Where $q$ is the average shear stress. Then

$$R_1 + R_2 + R_3 = q, (Q_1 + Q_2 + Q_3) = R$$

$$\therefore R = q, Q$$
Substituting Eq. (3), (4), (5) in Eq. (15)

\[ R = q, A \]  

(16)

Using Eq. (12), (13) and (14) in Eq. (16)

\[ R_1 = \frac{R}{A} \times Q_1 \]  

(17)

Generalising

\[ R_1 = \frac{R}{A} \times Q_i \]  

(18)

Now \( Q_i \) can be found from Eq. (3), (4) and (5) and Eq. (7), (8), (9) and (10). In this way, all the nodal loads are determined.

5.2 To determine the column shear

Since there is a hinge in the middle of the beam, the shear in column in any storey of the split frame will be equal. This is proved as follows: The terminal moment in any member will be represented by the three capital letters as \( MIJ \) which means, moment at \( I \) in the member \( IJ \). Now considering the top most storey and the top beam \( IJ \), it is known \( MIJ = MJI \)

(19)

Since there is a hinge in the middle.

When lateral load alone is acting in the frame, it is known that in any point

Sum of column moments = Sum of beam moments

Therefore, at joint \( I \)

\[ MIJ = MIE = MJI = MJF \]  

(20)

From which

\[ MIE = MJF \]  

(21)

Since the two column moments are equal, the illusion is that the shears in the left and right column hinge of the split frame will be equal. Hence, having determined \( R_i \) using Eq. (18), it is simply halved and placed at the two hinges as shears. Then, multiplying the shear and the lever arm gives the terminal moments. In side joint of the split frame, the beam moment is equal to the sum of two column moments meeting at the joint. In this way, all the terminal moments of beams and columns are found for all the split frames. To get back the original structure, all the split frames are added. In this process, the column moments get added up and the beam moments remain unaltered.

5.3 Illustrative Example

Considering Fig. 1, the numerical data are \( R = 10 \text{kN}, S = 10 \text{kN} \)

\[ l_1 = 20 \text{ m, } l_2 = 25 \text{ m, and } l_3 = 30 \text{ m.} \]
Solution:

From Eq. (7), (8), (9) and (10)

\[ A_1 = 0.5l_1 = 10 \text{ units} \]
\[ A_2 = 0.5l_1 + 0.5l_2 = 22.5 \text{ units} \]
\[ A_3 = 0.5l_2 + 0.5l_3 = 27.5 \text{ units} \]
\[ A_4 = 0.5l_3 = 15.0 \text{ units} \]

Check: \( A = A_1 + A_2 + A_3 + A_4 = 75 = l_1 + l_2 + l_3 = 75 \text{ m} \)

Now from Eq. (3), (4) and (5)

\[ Q_1 = A_1 + 0.5A_2 = 21.25 \text{ units} \]
\[ Q_2 = 0.5A_2 + 0.5A_3 = 25.00 \text{ units} \]
\[ Q_3 = 0.5A_3 + A_4 = 28.75 \text{ units} \]
\[ Q = Q_1 + Q_2 + Q_3 = 75.00 \text{ units} \]

From the above \( R_1 = Q_1 \times \frac{R}{A} = 21.25 \times \frac{10}{75} = 2.83 \text{ kN} \)
\[ R_2 = 25 \times \frac{10}{75} = 3.33 \text{ kN} \]
\[ R_3 = 28.75 \times \frac{10}{75} = 3.87 \text{ kN} \]

Check \( R_1 + R_2 + R_3 = R = 9.999 \approx 10 \text{ kN} \)

Similarly, the bottom storey is analysed. The rest of the procedure is shown in figure 2.
Then using the known information, in any joint, the sum of column moments is equal to the sum of beam moments, it is proved that the two column shears in the split frame are equal. In this way, the shear in the column is evaluated. Then, multiplying the shear and the lever arm gives the terminal moment. Computation of beam moments is straightforward, i.e., the sum of column terminal moments in any joint of the split frame gives the beam moment. Even though the split frame is unsymmetrical in geometry, it is symmetrical with regard to stress resultants because of the fact that a hinge is assumed in the middle of the beam. Thus, the computational effort is much reduced and the proposed method elucidates the analysis for a determination of shear in the various columns.

Table 1: Results of the Proposed Method and Comparison for Column Moments for Illustrative Example No.1

<table>
<thead>
<tr>
<th>Sl.No.</th>
<th>Member</th>
<th>Simplified portal method kNm</th>
<th>Improved portal method kNm</th>
<th>Proposed method kNm</th>
<th>Slope Deflection method kNm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>MAE</td>
<td>33.3</td>
<td>26.7</td>
<td>28.2</td>
<td>29.6</td>
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<tr>
<td>2</td>
<td>MEA</td>
<td>33.3</td>
<td>26.7</td>
<td>28.2</td>
<td>25.1</td>
</tr>
<tr>
<td>3</td>
<td>MFB</td>
<td>66.7</td>
<td>60.0</td>
<td>61.5</td>
<td>60.9</td>
</tr>
<tr>
<td>4</td>
<td>MFB</td>
<td>66.7</td>
<td>60.0</td>
<td>61.5</td>
<td>53.7</td>
</tr>
<tr>
<td>5</td>
<td>MCG</td>
<td>66.7</td>
<td>73.3</td>
<td>71.7</td>
<td>76.8</td>
</tr>
<tr>
<td>6</td>
<td>MGC</td>
<td>66.7</td>
<td>73.3</td>
<td>71.7</td>
<td>68.3</td>
</tr>
<tr>
<td>7</td>
<td>MDH</td>
<td>33.3</td>
<td>40.0</td>
<td>38.3</td>
<td>45.6</td>
</tr>
<tr>
<td>8</td>
<td>MHD</td>
<td>33.3</td>
<td>40.0</td>
<td>38.3</td>
<td>40.0</td>
</tr>
<tr>
<td>9</td>
<td>MEI</td>
<td>12.5</td>
<td>10.0</td>
<td>10.6</td>
<td>6.5</td>
</tr>
<tr>
<td>10</td>
<td>MIE</td>
<td>12.5</td>
<td>10.0</td>
<td>10.6</td>
<td>10.6</td>
</tr>
<tr>
<td>11</td>
<td>MFJ</td>
<td>25.0</td>
<td>22.5</td>
<td>23.1</td>
<td>18.3</td>
</tr>
<tr>
<td>12</td>
<td>MJF</td>
<td>25.0</td>
<td>22.5</td>
<td>23.1</td>
<td>24.8</td>
</tr>
</tbody>
</table>
5.4 Remarks on approximate and exact analysis

For a linearly elastic structure, exact analysis is one which satisfies both equilibrium and compatibility conditions. On the other hand, approximate methods used in lateral load analysis fulfill only the equilibrium requirement. In this work, the result of exact analysis by the slope – deflection method are presented for comparison. If the assumptions used in the approximate analysis regarding hinge and shear force or axial force coincide with that of the hinge positions and force conditions of the exact method then both the results will be alike. Since the approximate methods do not make use of member dimensions, it is unlikely that all the assumptions match with that of the exact solutions. Whenever congruence occurs, then the results will be same and in other places they will not. Slope – deflection, Moment Distribution, Kani’s method and FEM belong to the category of exact analysis. However, in the case of FEM, it is exact only when truss and beam elements are used in the particular problem. For other elements, it will be approximate. In FEM (being displacement method) compatibility condition is fully satisfied, but equilibrium of stresses at the node will not be met with fully and there will be error. This error will become negligible as finer and finer mesh is resorted to.

Table 2: Results of the Proposed Method and Comparison for Beam Moments for Example No.1

<table>
<thead>
<tr>
<th>Sl.No</th>
<th>Member</th>
<th>Simplified portal method kNm</th>
<th>Improved portal method kN.m</th>
<th>Proposed method kNm</th>
<th>Slope Deflection method kNm</th>
</tr>
</thead>
<tbody>
<tr>
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<td>MFE</td>
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<td>45.6</td>
<td>45.8</td>
<td>42.5</td>
</tr>
<tr>
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<td>MGF</td>
<td>45.8</td>
<td>45.6</td>
<td>45.8</td>
<td>41.3</td>
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<tr>
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<td>MGH</td>
<td>45.8</td>
<td>55.2</td>
<td>52.7</td>
<td>52.2</td>
</tr>
<tr>
<td>6</td>
<td>MHG</td>
<td>45.8</td>
<td>55.2</td>
<td>52.7</td>
<td>53.6</td>
</tr>
<tr>
<td>7</td>
<td>MIJ</td>
<td>12.5</td>
<td>10.0</td>
<td>10.6</td>
<td>10.6</td>
</tr>
<tr>
<td>8</td>
<td>MJI</td>
<td>12.5</td>
<td>10.0</td>
<td>10.6</td>
<td>10.0</td>
</tr>
<tr>
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<td>MJK</td>
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<td>12.5</td>
<td>14.8</td>
</tr>
<tr>
<td>10</td>
<td>MKJ</td>
<td>12.5</td>
<td>12.5</td>
<td>12.5</td>
<td>14.4</td>
</tr>
<tr>
<td>11</td>
<td>MKL</td>
<td>12.5</td>
<td>15.0</td>
<td>14.4</td>
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<tr>
<td>12</td>
<td>MLK</td>
<td>12.5</td>
<td>15.0</td>
<td>14.4</td>
<td>18.5</td>
</tr>
</tbody>
</table>

MLK means moment at L in the member LK

5.5 Recapitulation

The two widely used approximate methods for the lateral load analysis of plane frames are the portal and cantilever methods. The nonpareil feature of these methods is that they do not necessitate the information regarding member cross sectional dimensions for the analysis.
The portal method has two avenues. The first one is a simplified version which is too plain giving rise to high discrepancy in the stress resultants. This deficiency is rectified in the second version known as improved portal method which is based on pro rata length of the columns. In this paper, an alternative procedure known as split frame method is put forward using the same data as the other two methods. The results of the proposed method are in harmony with the solution of the improved portal method, which manifests its efficacy. The method does not involve much computational effort and is quite intelligible, transparent and unqualified in concept. In fine, it is opined that the proposed method is a supplement to the improved portal method.

6. References


