Interaction of compression and shear in transversely stiffened web
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ABSTRACT
Thin webs are stiffened transversely to protect it from buckling. In combined loadings of bending \((M)\), shear \((V)\) and axial compression \((P)\), dominating actions for buckling is axial compression \((P)\) and shear \((V)\). The present study is aimed towards understanding Limit State behaviour of transversely stiffened web of girder in pure axial compression \((P)\), pure shear \((V)\) and hence in combined action of compression and shear. The span to depth ratio \((\text{aspect ratio, } a/b)\) of web panel plays important role in this regard. Further the \(V–P\) interaction is also studied & verified. Numerical analysis by FEM for pure axial compression and pure shear are validated by classical analysis.

Keyword: Buckling, stiffened web, axial compression, shear, interaction of compression and shear.

1. Introduction
In case of beam of I section, if depth of web increases as in case of plate girder, it becomes sensitive for local web crippling and web buckling due to diagonal compression by shear and compression due to bending. Many times element is subjected to combined action of bending, shear and axial compression \((M-V-P)\). Various literatures are available on interaction of \((M-P)\) as well as \((M-V)\). The interaction of shear and axial compression \((V-P)\) is less attended even it is not considered in design provisions.

Interaction of Shear \((V)\) and Axial Compression \((P)\) is more critical at pinned base of column. Many times pinned base is preferred over fixed base due to economy in foundation cost. It is also important in case of Structural Steel member of plane portal frame in which all three actions such as Moment \((M)\), Shear \((V)\) and Axial Compression \((P)\) exists.

In the present study elastic behaviour of plate under inplane actions is considered. Elastic behaviour or elastic buckling strength is influenced by aspect \((a/b)\) ratio, boundary conditions and depth to thickness ratio (web slenderness ratio). For the present study web panel of stiffened plate girder is considered, for which the web panel is assumed as simply supported on all sides. Numerical case studies are carried out for web under the combined action of shear \((V)\) and axial compression \((P)\) for various aspect ratios and interaction equation is validated.

2. Classical theories
Classical plate theories are established by earlier researchers for elastic buckling of plate under inplane action of pure compression and pure shear.
2.1 Buckling of web panel under uniaxial compression (SSSS)

Considering a rectangular plate as shown in Figure -1, compressed in it’s middle plane by forces uniformly distributed along the sides \( x = 0 \) and \( x = a \). Let the magnitude of this compressive force per unit length of the edge be denoted by \( N_x \). By gradually increasing \( N_x \) and using equilibrium of the compressed plate, critical load at buckling is given by,

\[
N_{x,cr} = \frac{\pi^2 D}{b^2} \left( \frac{mb}{a} + \frac{a}{mb} \right)^2
\]

Where ‘m’ is the one buckling mode along X direction. With the change in aspect ratio, number of buckling mode is changing. Thus, the buckling coefficient ‘k’ will be given as,

\[
k = \left( \frac{mb}{a} + \frac{a}{mb} \right)^2
\]

The critical load, with \( m = 1 \), can be finally represented in the following form –

\[
(N_{x})_{cr} = \frac{\pi^2 D}{b^2} \left( \frac{b}{a} + \frac{a}{b} \right)^2
\]

Where \( D \) is the flexural rigidity of plate.

Considering ‘t’ as the thickness of plate, the critical value of the compressive stress is

\[
\sigma_{cr} = \frac{(N_{x})_{cr}}{t} = \frac{k \pi^2 E}{12(1-\mu^2)} \frac{t^2}{b^2}
\]

(Eq. 4)

For a given ratio \( a / b \) the values of coefficient \( k \) are as shown in Table 1.

<table>
<thead>
<tr>
<th>( a/b )</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
<th>1.1</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
<th>1.41</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>8.41</td>
<td>6.25</td>
<td>5.14</td>
<td>4.53</td>
<td>4.20</td>
<td>4.04</td>
<td>4.00</td>
<td>4.04</td>
<td>4.13</td>
<td>4.28</td>
<td>4.47</td>
<td>4.49</td>
</tr>
<tr>
<td>( \sigma_{cr} )</td>
<td>157.59</td>
<td>116.81</td>
<td>96.07</td>
<td>85.01</td>
<td>78.79</td>
<td>76.03</td>
<td>74.65</td>
<td>76.03</td>
<td>77.41</td>
<td>80.17</td>
<td>83.63</td>
<td>84.32</td>
</tr>
</tbody>
</table>

Table 1: Values of Factor \( k \) for Uniformly Compressed, Simply Supported Rectangular Plates and \( \sigma_{cr} \) in MPa for \( E = 2 \times 10^5 \) MPa, \( t/b = 100 \), \( \mu = 0.3 \)
2.2 Buckling of web panel under the action of uniform shear (SSSS)

Considering a rectangular plate as shown in Fig. 2, simply supported, and subjected to the action of shearing forces N_{xy} uniformly distributed along the edges.

![Figure 2: Plate Subjected to Uniform Shear](image)

The critical value of shearing stress, \( \tau_{cr} \), at which buckling of the plate occurs is determined using the energy method. Thus critical shear stress at buckling is given by

\[
\tau_{cr} = k_v \frac{\pi^2 D}{b^2 t}
\]

(5)

Where \( k_v \) is the buckling coefficient, which depends on the ratio \( a/b \). As per the classical solution, for a square plate \( k_v = 9.34 \). For the various ratios \( a/b \), the coefficient \( k_v \) is as shown in Table 2.

<table>
<thead>
<tr>
<th>( a/b )</th>
<th>1.0</th>
<th>1.2</th>
<th>1.4</th>
<th>1.5</th>
<th>1.6</th>
<th>1.8</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>9.34</td>
<td>8.0</td>
<td>7.3</td>
<td>7.1</td>
<td>7.0</td>
<td>6.8</td>
<td>6.6</td>
<td>6.1</td>
<td>5.9</td>
<td>5.7</td>
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</tbody>
</table>

3. FEM analysis

3.1 Buckling of web panel under uniaxial compression (SSSS plate)

The critical elastic buckling force (N_{xcr}) for a thin plate in pure uniaxial compression is determined by FEM analysis using ANSYS. An 8 nodded, shell 93 element with 6 degrees of freedom is used for analysis in ANSYS. A plate of size 1.44 X 1.2 (a/b ratio 1.2) is modelled (Fig. 3) and by applying uniaxial pressure eigen-buckling analysis is performed. The buckled shape of the plate is as shown in figure 3.

Buckling coefficient ‘k’ by classical theory is 4.13 and by FEM analysis it is found to be 4.088. Thus an error observed is only 1.00 %
3.2 Buckling of web panel under uniform Shear (SSSS plate)

The critical elastic buckling force \((N_{xy,cr})\) for a thin plate in pure shear is determined by FEM analysis using ANSYS. A 4 noded, shell63 element with 6 degrees of freedom is used for analysis in ANSYS. A plate of size 1.44 X 1.2 \((a/b\) ratio 1.2) is modelled (Figure 4) and by applying uniform shear force eigen-buckling analysis is performed. The buckled shape of the plate is as shown in figure 4.

Buckling coefficient ‘k’ by classical theory is 8 and by FEM analysis it is found to be 8.088. Thus an error observed is only -1.04%.

4. Case Studies for interaction of P & V

In this case study, the influence of web panel dimensions i.e. ratio \((a/b)\) on interaction of Axial Compression \((P)\) and Shear \((V)\) for Stiffened web of Plate Girder is observed.

For shear and longitudinal compressive direct stress, the interaction curve for all span to depth ratios is as given by
\[
\left( \frac{\sigma_x}{\sigma_{cr}} \right) + \left( \frac{\tau}{\tau_{cr}} \right)^2 = 1 \tag{6}
\]

Where,

\( \sigma_x \) = Actual axial stress in compression

\( \sigma_{cr} \) = Critical axial stress in compression

\( \tau \) = Actual shear stress

\( \tau_{cr} \) = Critical shear stress

For parametric study, following is the formulation considered,

As per classical study, critical axial compressive stress is given as

\[
\sigma_{x,cr} = \left( \frac{K\pi^2D}{b^3t} \right)
\]

Where D is flexural rigidity of plate,

\[
D = \left( \frac{E t^3}{12(1-\nu^2)} \right)
\]

As well critical shear stress is given as

\[
\tau_{cr} = \left( \frac{K_v \pi^2D}{b^3t} \right)
\]

As per IS 800: 2007 provisions, resistance to shear buckling needs to be considered when the ratio \( d/t_w > 67\sqrt{\frac{K_v}{5.35}} \), where, ‘d’ is the depth of web with stiffeners, \( t_w \) is the thickness of web and \( \varepsilon = \frac{250}{f_y} \).

Shear strength of web with stiffeners as governed by buckling may be evaluated as

\[
V_{cr} = A_v \tau_b
\]

Further \( \tau_b \) is the shear stress corresponding to the web buckling, which depend upon non dimensional web slenderness ratio for shear buckling stress, \( \lambda_{sw} \).

\[
\lambda_{sw} = \sqrt{\frac{f_{yw}}{(\sqrt{3} \tau_{cr})}}
\]

Thus, in the above mentioned interaction equation, Eq.(6), \( \tau_b \) is used in place of \( \tau_{cr} \).

As the present study is related to the web of stiffened plate girder the boundary conditions are considered as all edges simply supported (ssss). Further, from the classical theory knowing that the transition of buckling mode from number of buckling modes one to two is at \( a/b = 1.41 \), for plate subjected to pure axial compression, the parametric study is limited to the ratio \( a/b = 1.41 \).

Now, assuming

\( b = 1200 \, \text{mm}, \, t = 10 \, \text{mm}, \, f_{yw} = 250 \, \text{N/mm}^2, \, \nu = 0.3 \), buckling coefficient K as per table 1, buckling coefficient \( k_v \) using equation
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\[ K_v = 4.0 + 5.35/(a/b)^2 \] for \( a/b < 1.0 \)
\[ K_v = 5.35 + 4.0/(a/b)^2 \] for \( a/b \geq 1.0 \)

\( \tau_b \), the shear stress corresponding to the web buckling is determined for the various \( a/b \) ratios and then interaction is plotted for non dimensional ratios of \( \sigma_x / \sigma_{x,cr} \) and \( \tau / \tau_b \).

The results are tabulated in table 3 & 4, and interaction is shown in figure 5.

**Table 4**: Results for \( (a/b = 0.8) \)

<table>
<thead>
<tr>
<th>( \sigma_x / \sigma_{x,cr} )</th>
<th>( (\tau / \tau_{cr})^2 )</th>
<th>( \tau / \tau_{cr} )</th>
<th>( \tau )</th>
<th>( \lambda ), cl.8.4.2.2a, IS 800: 2007</th>
<th>( \tau_b ), cl. 8.4.2.2a, IS 800: 2007</th>
<th>( \tau / \tau_b )</th>
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<tbody>
<tr>
<td>0.00</td>
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<td>1.00</td>
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<td>125.333</td>
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<td>0.90</td>
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<td>0.965</td>
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<td>0.965</td>
<td>125.333</td>
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</table>

**Table 5**: Combined results for different \( (a/b) \) ratios of Web

<table>
<thead>
<tr>
<th>( \sigma_x / \sigma_{x,cr} )</th>
<th>( \tau / \tau_{cr} )</th>
<th>( \tau )</th>
<th>( \lambda ), cl.8.4.2.2a, IS 800: 2007</th>
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<td>125.333</td>
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5. Discussions and conclusions

Based on parametric studies of web panel dimensions i.e. ratio \( a/b \) of Stiffened web of the girder following broad conclusions are drawn

1. Classical solution for the stiffened web panel in pure uniaxial compression and in pure shear are compared with numerical solutions obtained by FEM. Using eight & four noded shell element in ANSYS, for pure compression and pure shear the error at the first buckling mode is found to be 1.00 % and -1.04 % respectively for the practical case of stiffened web panel with ratio \( a/b = 1.2 \)

2. In case of Stiffened web panel, boundary conditions as laid down by IS 800: 2007 are considered, taking all edges of web panel formed by two successive transverse stiffeners and web-flange plate connection as Simply Supported. With increase in web panel ratio \( a/b \) it is observed that the strength in shear decreases for any given value of axial compressive stress.

3. It is observed that for ratio \( a/b \) beyond 1.3 the ratio of shear strength remains constant.
   Thus interaction of Shear (V) and Axial Compression (P) for stiffened web panel of \( a/b = 1.3 \) onwards remains unchanged. This means interaction is independent of ratio \( a/b \) beyond the value 1.3.

4. The above conclusion shows that maximum spacing of transverse stiffeners be kept upto 1.3 times depth of girder.

6. References

1. AliniaÅ M. M., Habashi H.R., Khorram A., (2009), Non linearity in the postbuckling behaviour of thin steel shear panels, Thin-Walled Structures,47, pp 412-420

2. Batdorf S. B. and Manuel Stein (1947), Critical combination of shear and direct stress for simply supported rectangular flat plates, National advisory committee for
Performance of Non-Linear elastomeric Base-Isolated building structure
Gomase O.P, Bakre S.V


5. Elbridge Z. Stowell and Edward B. Schwartz (1943), Critical stress for an infinitely long flat plate with elastically restrained edges under combined shear and direct stress, National advisory committee for aeronautics (NACA) – Wartime Report, Article No. (3K13)


11. Manuel Stein and John Neff (1947), Buckling stresses of Simply Supported Rectangular flat plates in shear, National advisory committee for aeronautics (NACA), Technical Note No. (1222).

12. Piscopo V., (2010), Buckling analysis of rectangular plates under the combined action of Shear and Uniaxial Stresses, World Academy of Science, Engineering and Technology, 70, pp 547-554.


17. Shengming Zhang, Imtaz Khan, (2009), Buckling and ultimate capability of plates
and stiffened panels in axial compression, Marine Structures, 22, pp 791-808

