Mass distribution of multiple tuned mass dampers for vibration control of structures
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ABSTRACT
The mass distribution is one of the important parameter for MTMD (Multiple Tuned mass damper) for reducing effectively dynamic response of a main system. By controlling mass distribution along with other parameters like damping ratio, frequency range, number of dampers, response of the main system can be controlled. Different mass distributions like parabolic mass distribution and bell-shaped mass distribution are studied and its effect on controlling the dynamic response of the system is reported. To increase the effectiveness of the MTMD system, modified parabolic mass distribution and modified bell shaped mass distribution by skewing the mass distribution is proposed. Optimum parameters for MTMD with optimum mass distribution and main system damping varied as 2% and 5% are presented. Among the various mass distributions proposed in the present study Modified bell-shaped mass distribution for MTMD is superior. It was found to be more promising in terms of reducing dynamic response of structure making it more flat and increasing bandwidth of flat region. Also, lower values of damper damping associated with this mass distribution makes system more workable.

Keywords: TMD, MTMD, mass distribution, skew mass distribution, DMF, optimum parameter.

1. Introduction
Increased safety levels and stringent performance requirements lead to control of vibration of structures subjected to dynamic loads. TMD (Tuned mass damper) is a modular device composed of mass, spring and dashpot which are attached to a vibrating main system, in order to suppress undesirable vibrations at a particular frequency. TMD itself is a single-degree-of-freedom (SDOF) resonant system, which adds a mode of vibration to the main structure. Since the natural frequency of the damper is tuned to a frequency near to the natural frequency of the main system, which needs to be damped, the vibration of main system causes the damper to vibrate in resonance, dissipating the vibration energy through the damping in TMD. Use of more than one TMD has been proposed to increase the robustness of the vibration control system to various uncertainties in the structure. Iwanami and Seto (1984) have shown that two TMDs are more effective than single TMD. Zuo and Nayfeh (2006) found that a two-DOF absorber can attain better performance than the optimal SDOF absorber, even for the case where the rotary inertia of the absorber tends to zero. However, the improvement was not significant. Multiple tuned mass dampers with distributed natural frequencies were proposed by Xu and Igusa(1992) and studied by Yamaguchi and Harnpornchai (1993) Abe and Fujino (1994), Jangid(1995) and Zuo and Nayfeh(2004) . The basic configuration of MTMD consists of large number of small oscillators whose frequencies are distributed around the natural frequency of controlled mode.
of main structure. It was shown in above studies that the optimally designed MTMDs are more effective and robust than an optimally designed single TMD. Varadarajan and Nagarajaiah (2003) investigated the effectiveness of a novel semi-active variable stiffness tuned mass damper (SAVS-TMD) for response control of wind excited tall benchmark building and reported its effectiveness. Lewandowski R. and Grzymislawksa J (2009) reported effective use of MTMD for controlling wind induced vibrations for frame building, where excitation forces are treated as random forces.

Yamaguchi and Harpornchai (1993) in their study considered mass ratio and damping ratio for all TMDs as constant and have shown that the response of the system improves considerably for optimum parameters of dampers as compared to single TMD and also the robustness of the system increases. They have also shown that for number of TMDs, \( n = 21 \) gives almost the same response curve as \( n = 25 \). Igusa and Xu (1994), while discussing optimal design of multiple TMDs considered mean square response of the main oscillator as performance function and total mass of TMDs as constraint which is computed in terms of mass density. The study shows that curve plotted for amplitude of mass density as against natural frequency assumes shape of half ellipse and area under this curve is equal to total mass of dampers. Park and Reed (2001) studied effect of uniform and linear variation of mass distribution of MTMD on performance of system subjected to harmonic excitation. They concluded that uniform mass variation gives more robustness to the system and at the same time improves the performance.

Above reported work shows that the mass distribution is an important parameter for MTMD and the structural response can be effectively controlled using proper mass distribution. The objectives of present study are (i) to study effect of a mass distribution which is a combination of linear and uniform mass distribution (which is closer to parabolic mass distribution) for controlling dynamic response of the system supported by MTMD and subjected to harmonic excitation, (ii) skew mass distribution ie placing the mass where it is required most to flatten the response curve is proposed and to study its effect for controlling dynamic response of system, (iii) To study the effectiveness of modified bell shaped mass distribution, and (iv) For the mass distribution which yields the best possible result, search the optimum parameters for MTMD namely damper damping, tuning frequency, band width to reduce displacement DMF(Dynamic magnification factor).

2. Structural model

For the Structural model of the system with MTMD (Figure 1) the Equation of motion for the analytical model in matrix form is

\[
\ddot{X} + C \dot{X} + KX = F
\]  

(1)

where the vector \( X \) is the displacement vector which consists of the displacement of the structure \( x_s \), and the displacement of \( k^{th} \) TMD as \( x_k (k=1,2,\ldots, n) \) that is

\[
\{X\} = \{x_s, x_1, x_2, \ldots, x_n\}^T
\]  

(2)

The mass matrix, damping matrix and stiffness matrix is as given below

\[
[M] = \text{diag}[m_s, m_1, m_2, \ldots, m_n]
\]  

(3)
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\[
[C] = \begin{bmatrix}
    c_1 + \sum c_j & -c_1 & -c_2 & \cdots & -c_n \\
    c_1 & 0 & 0 & \cdots & 0 \\
    c_2 & 0 & 0 & \cdots & 0 \\
    \vdots & \ddots & \ddots & \ddots & \ddots \\
    c_n & \cdots & \cdots & \cdots & 0
\end{bmatrix}_{\text{sym}}
\]

\[
[K] = \begin{bmatrix}
    k_1 + \sum k_j & -k_1 & -k_2 & \cdots & -k_n \\
    k_1 & 0 & 0 & \cdots & 0 \\
    k_2 & 0 & 0 & \cdots & 0 \\
    \vdots & \ddots & \ddots & \ddots & \ddots \\
    k_n & \cdots & \cdots & \cdots & 0
\end{bmatrix}_{\text{sym}}
\]

(4)

(5)

\[
\{ F \} = [ f_0 \ e^{iat}, 0, 0, \ldots, 0]^T
\]

(6)

Where, \( f_0 \) is the force amplitude and \( \omega \) is the frequency of the applied force.

The corresponding steady-state harmonic response of the system to the harmonic excitation will be \( \{ X \} = X(\omega) e^{iat} \). The \( X(\omega) \) indicates the amplitude vector of the steady-state response of the combined system which is expressed by

\[
X(\omega) = \left( \omega^2 [M] + i\omega [C] + [K] \right)^{-1} \{ 1 \} f_0
\]

(7)

**Figure 1:** Structural model of a main system with MTMD

The main system is idealized as a single lumped mass characterized by the stiffness, \( k_s \), the damping constant, \( c_s \), and the mass, \( m_s \). It is assumed that the damping in the main system is...
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of viscous type specified by the damping ratio, $\xi_i$ (i.e., $\xi_i = c_i / 2\sqrt{m_i k_i}$). Similar to the main system, the parameters of the $j^{th}$ TMD are the stiffness, $k_j$, the damping constant, $c_j$, and the mass, $m_j$. The main system and each TMD are modelled as a SDOF system under harmonic excitation so that the total degrees-of-freedom of the combined structural system are $n + 1$. The natural frequency, $\omega_j$ (i.e., $\omega_j = \sqrt{k_j / m_j}$) of the $j^{th}$ TMD is expressed by

$$
\omega_j = \omega_r \left[1 + \left( \frac{j - \frac{n+1}{2}}{\beta} \right) \frac{\beta}{n-1} \right]
$$

(8)

$$
\omega_r = \frac{\sum_{j=1}^{n} \omega_j}{n}
$$

(9)

$$
\beta = \frac{\omega_n - \omega_1}{\omega_r}
$$

(10)

Where, $\omega_r$ is the average frequency of all MTMD; and $\beta$ is the non-dimensional frequency band-width of the MTMD system.

The damping constant of the $j^{th}$ TMD is expressed as

$$
c_j = 2m_j \xi_r \omega_j
$$

(11)

Where, $\xi_r$ is the damping ratio kept constant for all the MTMD. Total mass of the MTMD system is expressed by the mass ratio defined as

$$
\mu = \frac{\sum_{j=1}^{n} m_j}{m_s}
$$

(12)

3. Effect of mass distribution

A plot of dynamic response of damped system with optimum parameters of dampers and with constant mass of dampers (Figure 2) shows that it is a two peak response. Den Hartog (1956) employed the method of equalizing peaks of response to arrive at optimum parameters of damped main system with single TMD. The main approach in present study is to transform two-peak response of a system with damping to a one-peak response and flatten out the peak by controlling the mass distribution along with other parameters of dampers like damping ratios, frequency range and the number of tuned mass dampers. Also there should be reduction in displacement DMF of the system.

The mass distribution is an important parameter for reducing and controlling the structural response of the system. From the work of Igusa and Xu(1994) and Park and Reed(2001) a mass distribution which is a combination of linear and uniform mass distribution is proposed for controlling response of the system. This can be closer to parabolic mass distribution. The reason for selecting parabolic mass distribution is that the shape of response curve is quite closer to the parabolic shape. A study is performed considering parabolic mass distribution as defined by Equation (13), where $m_j$ defines mass of $j^{th}$ damper and $a$ is any positive number.

$$
m_j = \left[1 + \left( \frac{n+1}{2} \right)^2 \right] a - \left( j - \frac{n+1}{2} \right)^2 \times \mu / m_s
$$

(13)
Due to the complexity of MTMD system an analytical closed form solution is not possible. Hence, the criteria selected for optimality is set of parameters that gives the minimization of the steady state displacement of the main system under harmonic excitation. The optimum parameters for dampers are frequency range, off-tuning frequency and damper damping. For the present study, numbers of dampers used are \( n = 11 \). Main system damping is assumed to be 2\% and mass ratio 1\%. The damping ratio is kept constant for all TMDs.

The study started with value of “\( a \)” chosen as 10 to give a parabolic mass distribution. The idea here is to have maximum mass at the centre so as to damp out central peaks. The entire distribution can be adjusted so as to damp out all the secondary peaks as well. One parameter is varied at a time keeping all other variables constant which yields minimum of maximum DMF. This value is then used for further cycles and next parameter is varied. Thus by searching set of parameters of dampers which gives minimum value of maximum DMF along with flat curve of structural response is chosen (Figure 3 (a) shows a typical parabolic mass distribution for number of dampers \( n = 11 \) and mass ratio as 1\% The total area under the curve will be total mass of dampers.).

**Figure 2:** Optimised response curve for constant mass of dampers

**Figure 3:** Parabolic and bell shaped mass distribution.
3.1 Effect of Band-Width

The optimum parameters are found out by numerically searching one parameter at a time keeping other parameters constant. For main system damping as 2% and number of dampers as 11, value of band-width is searched first such that minimum value of maximum DMF and maximum flat curve of structural response is obtained. Parameter $\beta$ measures the spacing of TMDs natural frequencies.

![Figure 4: Effect of band-width on dynamic response of structure](image)

The vibration control mechanism by MTMD with optimum frequency range is to widen the effective frequency region and to flatten the structural response curve in that region. It can be seen from the Figure 4, that the controlled structural response is transformed from a two peak response to a one peak characteristic with increasing frequency range of MTMD. As seen from Figure 4, larger value of $\beta$ increases DMF and reduces flat region of response curve. The frequency distribution of MTMD affects the response of structure over wider range and this widely ranged effect of MTMD reduces the structural response. There is therefore a value of the frequency range for which the response curve is flat for wide range of frequencies and peak is minimum and the value at which this occurs is $\beta = 0.15$. The minimum value of maximum DMF is 8.1642.

3.2 Effect of Damper Damping

The next important parameter which will reduce value of DMF is damper damping. Using the best searched value of band-width,$\beta$, as 0.15 and $a = 10$, value for damper damping is varied to search optimum value. It can be seen from the Figure 5 that small damping ratios give rise to large secondary peaks, caused by resonance in the TMD’s response which gives the maximum response of the structure. Where as the secondary peaks flatten out with increasing damping ratios leading to a single peak at natural frequency of the structure. This is then the optimum response of the structure. The maximum structural response however increases if the damping ratio is increased too much. This is because the response of each TMD is not significant in the case of very large damping ratio. Therefore there exists an optimum value
of the damping ratio for an MTMD-Structure system (with varying mass ratio) in much the same manner as single TMD.

![Figure 5](https://example.com/figure5.png)

**Figure 5**: Effect of damper damping on dynamic response of structure

The optimum value of damper damping was found as 0.018 (Figure 5). Though the curve obtained is flatter, there is not much reduction in the value of DMF. The parabolic mass distribution of dampers improves system response in terms of reduction in DMF to some extent and better response curve (Figure 5). But, the difference in value of DMF for constant mass of dampers as against parabolic mass distribution for dampers is not significant.

### 3.3 Effect of Placement of Mass

With the use of above values of parameters of dampers, namely band-width and damper damping, the search is carried out for value of parameter \(a\) which will lead to placement of mass of each damper, i.e., more mass allocated at centre in comparison to extreme TMDs or else flatter mass distribution. It is seen that for \(a = 3\) value of DMF reduces and at the same time better structural response curve is obtained. Now with value of \(a\) as 3, damper parameters, namely damper damping, band-width and off set frequency, are again searched to arrive at minimum value of maximum DMF and better response curve. It is seen that though the reduction in the value of DMF is 2.9%, the response curve obtained is much better (Figure 6). The minimum value of maximum DMF for optimum value of damper parameters and with parabolic mass distribution is 7.954 as against 8.198 for constant mass of dampers.

### 4. Skew mass distribution

It was observed that the typical response curve with optimum parameters for constant mass as well as symmetric parabolic mass distribution scheme is a two-peak response curve. The two peaks are of unequal heights. For symmetric parabolic mass distribution for dampers, response curve obtained is much flatter as compared to response curve with constant mass. The parabolic distribution gave rise to families of response curves that had a higher DMF to
the left of the natural frequency of the structure ($\frac{\omega}{\omega_s} < 1$). This suggests that instead of having a symmetrical distribution, we should use a skewed distribution with more mass associated with lower TMD frequencies.

A modified mass distribution scheme is proposed where mass is skewed to obtain flat peak response with minimum DMF. By skewing the mass, it is placed at position where it is required most to reduce the dynamic response of the system. A modified distribution of the form as given by equation (14) is proposed.

$$m_j = \left[ 1 + \left( \frac{n+1}{2} \right)^2 \right] a - \left( j - \frac{n+1}{2} \right)^2 \times \frac{\mu}{m_j} \times (\gamma(j))$$  \hspace{1cm} (14)

where

$$\gamma_j = \frac{\omega_j}{\omega_s}$$  \hspace{1cm} (15)

The sign of parameter $r$ and magnitude of parameter $r$ would skew the distribution to either side and its magnitude can be used to alter the degree of shift in distribution. The value of parameter $a$, which defines placement of mass in parabolic mass distribution is also one of the important parameter which may reduce value of DMF and make the curve flatter. This parameter is also searched and the best value $a = 3$ is used for further study. Once again the parameters were searched systematically and preliminary values for each parameter were arrived at, by choosing the one, which gave minimum value for maximum DMF and a flat response. The structural response with skew modified parabolic distribution with optimum damper parameters shows that though the value of DMF is reduced from 7.9545 to 7.9471, there is no significant advantage in terms of reduction in DMF but the curve obtained is much flatter over a frequency range.

**Figure 6**: Optimum response curve for parabolic mass distribution

A modified mass distribution scheme is proposed where mass is skewed to obtain flat peak response with minimum DMF. By skewing the mass, it is placed at position where it is required most to reduce the dynamic response of the system. A modified distribution of the form as given by equation (14) is proposed.
4.1 Skew Bell-shaped Mass Distribution

The skew modified parabolic distribution works well to give a reduced and a much flatter response curve. However, a completely flat peak response was not attained. This means that the idea of skewed mass distribution to achieve a symmetrical distribution is workable. A better mass distribution is needed to get a perfectly flat response curve. It can be seen that typical response of the system resembles bell shape. So, it was proposed to use a bell-shaped distribution that is similar to the response of the system. The option of skewing the mass towards one side is maintained since it yields better results.

Modified bell-shaped mass distribution as given by Equation (16) is proposed for further study.

\[
    dm_j = \left\{ \frac{a^3}{a^2 + \left( j - \frac{n + 1}{2} \right)^2} \right\} \times \mu / m_s \times (\gamma (j))
\]  

(16)

The sign of parameter \( r \) would skew the distribution to either side and its magnitude can be used to alter the degree of shift in distribution. (Figure 3 (b) shows a typical bell-shaped mass distribution for number of dampers \( n = 11 \) and mass ratio as 1 %). The preliminary values of parameters, namely band-width, damper damping, frequency ratio, for further study are the values obtained for skew modified parabolic mass distribution. With these set of values, the value of parameter, \( a \), is searched. As value of \( a \) increases the mass distribution approaches flatter mass distribution and hence response curve gives larger value of peak response. The best value obtained is \( a = 29 \). For value of \( a = 29 \) parameter, \( r \) is varied to arrive at best value of DMF and flatter response curve, as seen from Figure 7. The value of \( r \) used for further study is 1.05. With these values, again damper parameters were searched numerically for band-width, damper damping, frequency ratio, and values for each parameter were arrived at, by choosing the one, which gave minimum value for maximum DMF and a flatter structural response. The results show that a correct combination of parameters yields a symmetrical and flat peak response (Figure 8).

![Figure 7: Effect of skew bell-shaped mass on dynamic response of structure](image-url)
The study is further extended for number of dampers, \( n = 11 \) and main system damping as 2% and for different mass ratio varying from 1% to 10% with interval of 1%. The values of optimum damper parameters were searched and are reported for different mass ratios in Table 1.

The figure 9 shows response curve with optimum parameters plotted for mass ratio 4%, 6% and 8%. The figure shows that the response of the system can be controlled effectively by proper mass distribution and there is systematic reduction in DMF value as mass ratio increases. Similar study is carried out for main system damping as 5%. Table 2 reports values of optimum damper parameters for \( n = 11 \) and main system damping as 5% and for mass ratio varying from 1% to 10%. Using above approach optimum damper parameters were searched and reported in tables 1 and 2 for number of dampers as 21 and main system damping as 2% and 5%. Similar study can be conducted for number of dampers as 5 and 7 also.
Table 1: Optimum parameters for number of dampers, $n = 11$ and $21$, $\xi_r = 2\%$ for modified bell-shaped mass distribution

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<th>$r$</th>
<th>Damper damping $\xi_T$</th>
<th>Bandwidth $\beta$</th>
<th>Tuning Frequency $f$</th>
<th>DMF</th>
<th>$\mu$</th>
<th>$a$</th>
<th>$r$</th>
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Table 2: Optimum parameters for number of dampers, $n = 11$ and $21$, $\xi_r = 5\%$ for modified bell-shaped mass distribution

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5. Comparison of Optimum Curves

Optimum parameters of dampers attached to damped main system subjected to harmonic excitation are obtained by varying different parameters which affect performance of a damped main system with MTMD. The criterion for optimality was set of parameters which gives flat response curve and minimum value of maximum DMF. The damper parameters searched are bandwidth, damper damping and off-tuning frequency. The results of study conducted for number of dampers $n = 11, 21$ and main system damping as 2 and 5% with different mass distributions are discussed below. Figure 6 shows that parabolic mass distribution for dampers yields better response curve than that for constant mass of dampers as seen in Figure 2. Further, use of skew modified parabolic mass distribution over parabolic mass distribution gives reduction in DMF and yields better results. Effect of placing the mass where it is required most for improving response is achieved by skewing the mass. Flatter response curve with reduction in the value of DMF is achieved by use of skew modified bell-shaped mass distribution. The minimum value of maximum DMF is brought down from 8.198084 (for constant mass) to 7.93677, an improvement of about 3.18% for 2% damped main system, number of dampers as 11 and mass ratio 1%. Moreover, the response curve obtained is flatter over wide range of frequency thus making the system much robust. When the mass ratio increases from 1% to 10%, for main system damping as 2%, with skew modified bell-shaped distribution, reduction in DMF is of the order of 14.23%. Figure 9 shows response curve plotted for different mass ratio namely 4%, 6% and 8%. As mass ratio increases the curve is flatter over wide frequency range and proportionally value of DMF also reduces as mass ratio increases. But there is restriction for using higher mass ratio and generally for civil engineering purpose, value of mass ratio greater than 10% is not used.

Figure 10: Comparison of optimum parameters for bell-shaped mass vibration for main system damping as 2% and 5%, number of dampers $n=11$
Optimum parameters for modified bell-shaped mass distribution for main system damping as 2 and 5% and for number of dampers as 11 are plotted in Figure 10. Optimum damper damping is lesser for system with 5% damping than for system with 2% damping, i.e., more damper damping is required for lesser damped system whereas for 5% damping of main system more off-tuning is required than 2% damped main system. Optimum band-width value is more for 5% damped system than for 2% damped system. This means that over large range of frequencies, flatter response can be obtained when system damping is more. Similar trend is observed for number of dampers as 21 and system damping as 2% and 5%. As system damping increases from 2% to 5%, damper damping reduces with increase in mass ratio. Similarly, minimum value of maximum DMF also reduces for moderately damped system as number of dampers increases along with mass ratio. Dynamic response of system with optimised bell-shaped mass distribution and for mass ratio as 1%, 2% and 3% for number of dampers \( n = 11 \) and \( n = 21 \) are plotted for main system damping as 5% in Figure 11. The plot shows that as number of dampers increase the curve obtained is flatter over large range of frequencies.

**Figure 11:** Response curve with optimise bell-shaped mass distribution and for mass ratio with number of dampers \( n = 11 \) and 21.

### 5.1 Concluding remarks

The following conclusions can be drawn from the study carried out. The optimally designed MTMD with mass distributed so as to have a maximum mass at the centre is more effective in reducing structural vibrations than an optimally designed MTMD with a constant mass ratio.

1. To minimize the structural response and flatten the response considerably, it is necessary to use a skewed mass distribution scheme.
2. Skew modified parabolic distribution for dampers, proposed in the present study was found to improve the structural response considerably. Response curve changes from two
peaks to a single peak as desired although a perfectly flat peak response curve was not obtained. Also the width of the frequency region is increased.

3. The best distribution obtained in the present study is Skew modified bell-shaped mass distribution for dampers. This was arrived at by placing a higher mass wherever response was high. Therefore, the mass was skewed to the side which has a higher peak response. The bell-shaped distribution was found to be more promising both in terms of reducing the DMF and increasing the bandwidth of flat region.

4. Increase in band-width makes the system more robust over wide frequency range.

5. Off tuned MTMD system is more effective. A tuning frequency slightly less than one was found to give minimum value of maximum DMF. This is similar to skewing mass towards one side.

6. As the mass ratio is increased, the DMF value goes on decreasing. But due to practical limitations on the mass of the overall system, higher mass ratios are generally not used.

7. For system of dampers with modified bell-shaped mass distribution, lower value of optimum damper damping makes system more workable from fabrication point of view.

6. References


