Comparison of design wind speed using Extreme Value Distributions and IS 875 Procedures

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ABSTRACT

Extreme wind speed is of significance in design process to study the structural safety particularly for tall structures like cooling towers, stacks, transmission line towers etc. Gumbel and Frechet distributions are used to estimate the design wind speed for different return periods. Method of least squares (MLS) and Lieblein technique based on order statistics approach are applied for estimation of parameters of the distributions. Anderson-Darling ($A^2$) test is used for checking the adequacy of fitting of the method/distribution to the recorded wind speed data. The study shows that Gumbel distribution using MLS is found to be appropriate for estimation of design wind speed for Vadodara region. The paper presents that the values of estimated 3-sec average extreme wind speed adopting statistical distributions are compared with IS 875 procedures to arrive at a design parameter for the region under study.

Key words: Anderson-Darling, Frechet, Gumbel, Method of least squares, Order statistics.

1. Introduction

Wind engineering analyses the effect of wind in the natural and built environment and the damage due to the wind. For a structural engineer, the interest is in strong winds which may cause discomfort. With the structures becoming tall and slender the effect of wind on these structures is becoming critical. The distribution of wind speed is also important in determining serviceability of buildings. The basic wind speed is arrived by considering 3-seconds (sec) average wind speed at a height of 10m; and the estimated value of 1000-year (yr) return period is used to arrive at a design load, which the structure must withstand during its lifetime (Coelingh et al, 1996; Bivona et al, 2003; Celik, 2004). For arriving at these values, wind records over a period of time are observed and a statistical analysis is carried out.

In statistical theory, Generalized Extreme Value (GEV) distribution is identified as a family of continuous probability distributions that include EV1 (Gumbel), EV2 (Frechet) and EV3 (Weibull). As defined by the extreme value theorem in statistics, the asymptotic distributions of the extremes tend to converge on certain limiting forms for large sample; specifically to the double exponential form, or to two different single exponential forms. Since the extreme values of a random variable are invariably associated with the tails of its probability density function, the convergence of the distribution function of its extreme value to a particular limiting form will depend on the behaviour at tail end of the initial distribution in the direction of the extreme (Ang and Tang, 1984; Deaves and Lines, 1997; Cook, 2001). In view of the above, Gumbel and Frechet distributions are used for estimation of design wind speed.
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for Vadodara region. Due to the convergence of the recorded data series, Weibull distribution is not considered in the analysis.

As suggested by AERB (2008), analytical procedures like method of least squares (MLS) and Lieblein technique based on order statistics approach (OSA) are applied for estimation of parameters of the distributions. Anderson-Darling (A²) test is used for checking the adequacy of fitting of the method/distribution to the recorded wind speed data. The methodology adopted in determining the parameters of Gumbel and Frechet distributions for estimation of design wind speed is briefly described in the ensuing sections.

2. Methodology

The cumulative distribution functions of Gumbel and Frechet distributions are given by:

\[ F(W) = e^{-\left(\frac{W - \alpha_G}{\beta_G}\right)} \]
\[ F(W) = e^{-\left(\frac{W}{\beta_F}\right)^{-\lambda_F}} \]

where, \( \alpha_G \) and \( \beta_G \) are the location and scale parameters of Gumbel distribution. The estimates of extreme wind speed using Gumbel distribution (\( W_G \)) for different return periods (T) are computed from \( W_G = \alpha_G + Y_T \beta_G \) with \( Y_T = -\ln(-\ln(1-(1/T))) \). Similarly, \( \beta_F \) and \( \lambda_F \) are the scale and shape parameters of Frechet distribution.

Based on extreme value theory, Frechet distribution can be transformed to Gumbel distribution through logarithmic transformation. Under this transformation, the estimates of extreme wind speed using Frechet distribution (\( W_F \)) for different return periods are computed from \( W_F = \exp(W_G) \), \( \beta_F = \exp(\alpha_G) \) and \( \lambda_F = 1/\beta_G \) (IAEA, 2003).

2.1 Estimation of distribution parameters

OSA is applied for estimating the parameters of Gumbel and Frechet distributions of the series consisting of the largest values. In addition, MLS is applied for determining the parameters of Gumbel distribution. The procedure adopted for estimating the parameters using MLS and OSA are as follows:

2.1.1 Method of least squares

The recorded extreme wind speed data are arranged in an increasing order of magnitude by assigning rank of 1, 2, 3, ..., N. The non-exceedance probability for each data point \( W_i \) for rank \(^{i} \) is assigned by using the following relation:

\[ P_i = \frac{i}{N+1} \]

Using MLS, the distribution parameters can be obtained from the following equations:
\[
\beta = \frac{\left( \sum_{i=1}^{N} W_i \right)^2 - \left( \frac{\sum_{i=1}^{N} W_i^2}{\sum_{i=1}^{N} W_i} \right)}{\left( \sum_{i=1}^{N} W_i \ln(-\ln(P_i)) \right) - \left( \frac{\sum_{i=1}^{N} \ln(-\ln(P_i))}{\sum_{i=1}^{N} W_i} \right)}
\]

\[
\alpha = \frac{\sum_{i=1}^{N} W_i}{N} + \left( \frac{\sum_{i=1}^{N} \ln(-\ln(P_i))}{N} \right) \beta
\]

Here, \( \ln(-\ln(P_i)) \) defines the cumulative probability of non-exceedance for each \( W_i \). The standard error (SE) on the estimated wind speed is computed by:

\[
SE = \frac{\beta}{\sqrt{N}} \left( 1.1589 + 0.1919Y_\gamma + 1.1Y_\gamma^2 \right)^{1/3}
\]

### 2.1.2 Order statistics approach

The approach is based on the assumption that the set of extreme values constitutes a statistically independent series of observations. The OSA parameters of Gumbel distribution are given by:

\[
\alpha_G = r^*\alpha_M^* + r'\alpha_M' ; \quad \beta_G = r^*\beta_M^* + r'\beta_M'
\]

where \( r^* \) and \( r' \) are proportionality factors, which can be obtained from the selected values of \( k, n \) and \( n' \) using the relations \( r^* = kn/N \) and \( r' = n'/N \). Here, \( N \) is the sample size contains basic data that are divided into \( k \) sub groups of \( n \) elements each leaving \( n' \) remainders.

In OSA, \( \alpha_M^* \) and \( \beta_M^* \) are the distribution parameters of the groups and \( \alpha_M' \) and \( \beta_M' \) are the parameters of the remainders, if any. These can be computed from the following equations:

\[
\alpha_M^* = (1/k) \sum_{i=1}^{k} \alpha_{ni} S_i ; \quad \alpha_M' = \sum_{i=1}^{n} W_i
\]

\[
\beta_M^* = (1/k) \sum_{i=1}^{k} \beta_{ni} S_i ; \quad \beta_M' = \sum_{i=1}^{n} W_i
\]

where \( S_i = \sum_{j=1}^{n} W_{ij}, j=1,2,3,...,n \). The values of the weights \( \alpha_{ni} \) and \( \beta_{ni} \) are given in IAEA (2003). The standard error (SE) on the estimated wind speed is computed by:

\[
SE = \left[ \text{Var}(W_{\gamma}) \right]^{1/2} \text{Var}(W_{\gamma}) = r^*W_n + r'W_n'
\]

where, \( r^* = \left( \frac{kn}{N} \right) \) and \( r' = \left( \frac{n'}{N} \right) \). \( W_n \) and \( W_n' \) are defined by the general form as \( W_{\gamma} = \left( A_n Y_G + B_n Y_G + C_n \beta_n \right) \). The values of \( A_n, B_n, \) and \( C_n \) are given in IAEA (2003).
2.1.3 Confidence limit for extreme wind speed

The Mean+1SE (where Mean denotes the estimated extreme wind speed and SE the standard error) upper confidence limit will represent the value that will not be exceeded by 84.13% of the events having a desired return period. In the present study, the Mean+1SE values obtained from Gumbel (using MLS and OSA) and Frechet (using OSA) distributions are compared with the values obtained from IS 875 procedure to arrive at a design parameter for Vadodara region.

2.2 Goodness-of-Fit test

The $A^2$ statistics is defined by:

$$A^2 = \left(1 - \frac{1}{N}\right) \frac{1}{\sum_{i=1}^{N} (2i-1) \ln(Z_{i}) + (2N+1-2i) \ln(1 - Z_{i})}$$

(1)

For a given sample of N values, $Z_i=F(W_i)$, for $i=1,2,3,...,N$; and $W_1<W_2<...W_N$. The distribution of $A^2$ statistics doesn’t depend on $F(W)$, but on the set of N sample values. If the computed value is less than that of critical value at the desired significance level ‘η’ then the selected method/distribution is accepted to be adequate than any other method/distribution for modelling extreme wind speed data (D’Agostino and Stephens, 1986).

2.3 Concept of IS 875 procedure

Following IS 875 (2003) procedure, the basic wind speed ($W_b$) for the region is obtained and subsequently modified to account for different effects and get design wind speed ($W_z$) at height $z$(m) for the chosen class of structure. The relationship between $W_b$ and $W_z$ can be expressed by:

$$W_z = W_b k_1 k_2 k_3$$

(2)

where, $k_1$ is the probability-factor/risk-coefficient, $k_2$ is the terrain and height factor and $k_3$ is the topography factor. Value of $k_1$ for different classes and mean probable design life of structures can be computed from the equation given by:

$$k_1 = \frac{A - B \ln \left(- \frac{1}{NYR} \ln(1 - P_{NYR})\right)}{A + 4B}$$

(3)

Here, NYR is the mean probable design life (yr) of the structure, $P_N$ the risk level in NYR consecutive years, A and B appropriate coefficients for the basic wind speed zone.

3. Results and Discussions

3.1 Estimation of wind speed using extreme value distributions

By applying the procedures, as described above, a computer program was developed and used to estimate the 3-sec average extreme wind speeds for Vadodara. Hourly wind speed data for the period 1969-91 was used to derive the series of 3-sec average extreme wind speed; and further used to estimate the extreme wind speeds for different return periods adopting Gumbel (using MLS and OSA) and Frechet (using OSA) distributions. By using the
monogram on normalized wind speed (AERB, 2008), the recorded 3-sec average extreme wind is obtained by multiplying the factor of 1.52 with hourly data. Table 1 gives the estimates of 3-sec average extreme wind speeds for different return periods given by Gumbel and Frechet distributions for Vadodara region.

Table 1: 3-sec average extreme wind speed estimates for different return periods using Gumbel and Frechet distributions

<table>
<thead>
<tr>
<th>Return period (yr)</th>
<th>Estimated 3-sec average extreme wind speed (km/hr) using</th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Gumbel</td>
<td>Frechet (OSA)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>MLS</td>
<td>OSA</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>SE</td>
<td>Mean+1SE</td>
<td>Mean</td>
</tr>
<tr>
<td>2</td>
<td>56.8</td>
<td>4.5</td>
<td>61.3</td>
<td>58.2</td>
</tr>
<tr>
<td>5</td>
<td>77.6</td>
<td>7.6</td>
<td>85.2</td>
<td>70.5</td>
</tr>
<tr>
<td>10</td>
<td>91.4</td>
<td>10.2</td>
<td>101.6</td>
<td>78.6</td>
</tr>
<tr>
<td>20</td>
<td>104.6</td>
<td>13.0</td>
<td>117.6</td>
<td>86.4</td>
</tr>
<tr>
<td>50</td>
<td>121.7</td>
<td>16.6</td>
<td>138.3</td>
<td>96.5</td>
</tr>
<tr>
<td>100</td>
<td>134.6</td>
<td>19.2</td>
<td>153.8</td>
<td>104.1</td>
</tr>
<tr>
<td>200</td>
<td>147.4</td>
<td>22.0</td>
<td>169.4</td>
<td>111.7</td>
</tr>
<tr>
<td>500</td>
<td>164.2</td>
<td>25.7</td>
<td>189.9</td>
<td>121.6</td>
</tr>
<tr>
<td>1000</td>
<td>177.0</td>
<td>28.4</td>
<td>205.4</td>
<td>129.1</td>
</tr>
</tbody>
</table>

From Table 1, it may be noted that the 3-sec average extreme wind speeds given by Gumbel distribution using MLS is consistently higher when compared with the corresponding values obtained from Gumbel and Frechet distributions using OSA. Also, from Table 1, it may be noted that Frechet distribution using OSA gives higher estimates for return periods above 10-yr when compared with the corresponding values of Gumbel using OSA.

3.2 Analysis based on GoF test

A² statistics for Gumbel (using MLS and OSA) and Frechet (OSA) distributions using the series of 3-sec average extreme wind speed were computed from Eq. (1), and given in Table 2.

Table 2: Computed values of A² statistics

<table>
<thead>
<tr>
<th>Extreme wind data series</th>
<th>Computed values of A² statistics for</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gumbel</td>
<td>Frechet (OSA)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MLS</td>
<td>OSA</td>
<td>MLS</td>
</tr>
<tr>
<td>3-sec average</td>
<td>0.730</td>
<td>2.692</td>
<td>2.978</td>
</tr>
</tbody>
</table>

From Table 2, it may be noted that the computed values of A² statistics given by Gumbel and Frechet distributions using OSA are greater than the critical value of 0.757 at five percent level, and at this level, OSA is not found to be suitable for estimation of design wind speed. Also, from Table 2, it may be noted that the computed value of A² statistics given by Gumbel distribution using MLS is less than the theoretical value of 0.757; and is a good choice for modelling 3-sec average extreme wind speed for Vadodara region.
3.3 Estimation of design wind speed using IS 875 procedures

Based on wind speed map given in IS 875, the basic wind speed for Vadodara region is found to be 50 m/s; and the coefficients of A and B corresponding to the basic wind speed is 88.8 and 22.8 respectively. Since Vadodara is considered to be a Terrain Category 1 i.e. exposed open terrain with few or no obstructions and in which the average height of any object surrounding the structure is less than 1.5 m, the values of $k_2$ and $k_3$ are considered as 1.05 and 1.00 respectively. The values of $k_1$ for different return periods such as 50-yr, 100-yr, 200-yr, 500-yr and 1000-yr are computed from Eq. (3). By using the values of $W_b$, $k_1$, $k_2$ and $k_3$, the values of design wind speed for different return periods, at a standard height of 10 m, are computed from Eq. (2). Table 3 gives a comparison of 3-sec average extreme wind speed estimates obtained from statistical and IS 875 procedures.

<table>
<thead>
<tr>
<th>Return period (yr)</th>
<th>IS 875</th>
<th>Estimated 3-sec average extreme wind speed (m/s) using</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Gumbel (OSA)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MLS</td>
</tr>
<tr>
<td>50</td>
<td>52.5</td>
<td>33.8</td>
</tr>
<tr>
<td>100</td>
<td>56.7</td>
<td>37.4</td>
</tr>
<tr>
<td>200</td>
<td>60.9</td>
<td>40.9</td>
</tr>
<tr>
<td>500</td>
<td>67.2</td>
<td>45.6</td>
</tr>
<tr>
<td>1000</td>
<td>71.9</td>
<td>49.2</td>
</tr>
</tbody>
</table>

From Table 3, it may be noted that the Mean+1SE (where Mean denotes the estimated wind speed and SE the standard error) values of 3-sec average extreme wind speeds for 50-yr, 100-yr, 200-yr, 500-yr and 1000-yr given by MLS of Gumbel distribution are less than the corresponding values obtained from IS 875 procedure.

Based on GoF test results, Gumbel distribution using MLS is found to be appropriate for estimation of extreme wind speed for different return periods. From the results of the data analysis, it may be noted that the percentages of variation on estimated Mean+1SE values of 3-sec average extreme wind speed with reference to design wind speeds corresponding to 50-yr, 100-yr, 200-yr, 500-yr and 1000-yr are about 27%, 25%, 23%, 22% and 21% respectively. The study suggested that the Mean+1SE value of 3-sec average extreme wind speed of 57.1 m/s (205.4 km/hr) related to 1000-yr return period may be adopted for design purposes in Vadodara region.

4. Conclusions

The paper presented a computer aided procedure for estimation of 3-sec average extreme wind speed for Vadodara region adopting Gumbel (using MLS and OSA) and Frechet (using OSA) distributions. IS 875 procedure was used to determine the design wind speed using basic wind speed for the region and the results were compared with the corresponding values obtained from statistical distributions. A$^2$-test results showed that the Gumbel distribution using MLS is found to be appropriate for estimation of extreme wind speed for different return periods. The results also showed that the percentages of variation on Mean+1SE values of the estimated 3-sec average extreme wind speed, with reference to design wind speed...
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5. References


