Hydrodynamic flood routing considering piedmont zone

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ABSTRACT

Hydrodynamic models are generally used for routing flood in river. Sometimes, solution of hydrodynamic equation carried out without considering some important aspects, like existence of piedmont zone in the computational domain. It has been observed that a significant amount of flow may infiltrate into the ground due to presence of piedmont zone in the river. A hydrodynamic flood routing model capable of handling the effect of a recharge zone in a river is presented in this paper. One dimensional continuity and momentum equation of unsteady free surface flow, coupled with Green-Ampt infiltration model has been used as governing equations. Second order finite difference explicit scheme is used for solution of the governing equation. Possible effect of piedmont zone on flood routing is analyzed by applying the model in a hypothetical non-prismatic river reach. Computational result obtained by considering the recharge zone is compared with the result without the recharge zone. From this canvass it has been observed that the hydrograph at downstream of the piedmont zone attenuate significantly because of the presence of recharge zone in the river. Similar effect has been also noticed in the water surface elevation. Results are found to be quite sensitive to hydraulic conductivity of the piedmont zone and therefore precise estimation of hydraulic conductivity is essential for reliable result.

Keywords: Piedmont zone, hydrodynamic model, Green-Ampt model, hydraulic conductivity.

1. Introduction

With the development of efficient numerical algorithm and the advent of high speed digital computer, solution of unsteady one dimensional and two dimensional hydrodynamic equation is becoming quite common for addressing various unsteady flow problem. Routing of flood hydrograph through a non-prismatic river reach is one such problem that has drawn attention of researchers since last century. Simulation of unsteady open channel flow by solving the Saint-Venant equations is an essential step for many applied problems such as flood forecasting, dam break analysis and watershed modeling. Keskin and Agiralioglu (1997) developed a one dimensional flood routing model using saint-Venant equation. Mohammad et al. (2006) developed unsteady flow model and he had used differential quadrature method for simulation of the unsteady flow. Zhang and Shen (2007) developed steady and unsteady flow model for channel networks. Fang et al. (2008) developed One-dimensional numerical simulation of non-uniform sediment transport under unsteady flows using Saint-Venant equations. The Saint-Venant equation is not amenable to analytical solution except for a few special cases. They are partial differential equation that, in general, must be solved using numerical methods. Various researchers used different finite difference scheme for solution of the Saint-Venant equation (Fennema et al. 1990, Rashid et al. 1995, Ramesh et al. 2000).
The unsteadiness in a river reach is caused by sudden change in the inflow due to occurrence of heavy rain in the upstream, breaches occurred in the embankment system or due to the collapse of a river valley structure. Existence of a recharge zone (piedmont zone) in a river can influence the flood movement significantly. Therefore influence of piedmont zone should be considered while assessing flood hydrograph at its downstream through mathematical modelling. Existence of such piedmont zone has been noticed in some of the Himalayan tributaries of Brahmaputra River. Goswami et al. (1996) identified the piedmont zone in the Assam. This paper presents a hydrodynamic model for routing flood hydrograph through a natural channel having piedmont zone. For developing the model component for free surface flow, Saint-Venant equation is used. The effect of recharge zone in the downstream hydrograph is considered and an inference is drawn in this paper.

Various researchers developed different infiltration model based on Green-Ampt method. Mein and Larson (1973) developed an infiltration model to simulate infiltration into a homogeneous soil under steady rainfall. They considered two distinct conditions, pre-ponding and post-ponding in their model. After that Chou (1978) further extended the Green-Ampt method to calculate infiltration in a homogeneous soil under an unsteady rainfall. Dagan and Bresler (1983) studied the appropriateness of Green-Ampt model for predicting water movement in homogeneous unsaturated soil. Govindaraju et al. (1995) used Green-Ampt model for analyzing one-dimensional convective transport in unsaturated soil. The Green-Ampt model for infiltration into homogeneous soils predicts a monotonically decreasing infiltration rate and a wetting front that initially advances as the square root of time. Infiltration in heterogeneous soil had been studied by different researchers (Childs and Bybordi 1969, Beven 1984, Selker et al. 1999 and Voller 2010). Bateman et al. (2010) used Green-Ampt method to test the effect of infiltration in a flooding process. In general, the height of the water surface is neglected in the Green-Ampt model. However, in their work they had considered the height of water surface as it was not negligible.

To the best of our knowledge this is the first attempt to solve unsteady flow in a river considering existence of a recharge zone in it. To develop a simple but logical mathematical formulation to represent ground water recharge from the river bed, we have applied Green-Ampt model with some logical assumption drawn from practical point of view.

2. One dimensional mathematical model

The proposed model has two components: 1) 1-D unsteady flow model for free surface flow computation, 2) infiltration model for computing time dependent infiltration rate and total infiltration

2.1 One dimensional unsteady flow model

One dimensional continuity equation and fully dynamic form of the momentum equation in non-conservation form has been used as a governing equation for solution of unsteady flow in a hypothetical non-prismatic rectangular river reach considered in this study. The continuity and momentum equation considering the piedmont zone in the river are given below in Eq. (1), Eq. (2)

\[
\frac{\partial A}{\partial t} + \frac{\partial q}{\partial x} + q_i = 0
\]  (1)
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\[
\frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{q^2}{A} \right) + gA \left( \frac{\partial y}{\partial x} - s_o + s_f \right) + m_l = 0
\]

\[s_f = \frac{v^2 n^2}{R^{4/3}}\]

Where \( q \) is the discharge, \( A \) is the cross sectional area of the river, \( y \) is the water depth, \( g \) is acceleration due to gravity, \( s_o \) is bed slope, \( s_f \) is friction slope, \( R \) is the hydraulic depth, \( n \) is the manning’s roughness coefficient, \( q_l \) is the lateral outflow per unit length of the river, \( m_l \) momentum loss per unit length of the river.

2.2 Infiltration model

Green-Ampt model as used by Bateman et al. (2010) for calculating infiltration in a flooded area has been used for computing infiltration component. In general, height of the water surface is neglected in Green-Ampt model. In the present study height of water surface cannot be neglected, as this becomes quite significant during flood time. The infiltration equation used for the recharge in the river bed can be written as,

\[
dF = -k_s \frac{h_0 + L(t) - h_f}{L(t)}
\]

The change in length of the moving wetting front is given by the following equation

\[
\frac{dL_f(t)}{dt} = -k_s \frac{h_0 + L(t) - h_f}{(\theta_s - \theta_i)L(t)}
\]

Where \( F \) is the cumulative infiltration, \( h_0 \) is the initial water surface height, \( L(t) \) is the length of the moving wetting front, \( h_f \) is the dry suction head, normally negative value, \( \theta_s, \theta_i \) are the saturated and initial soil moisture content, \( K_s \) is the saturated hydraulic conductivity.

3 Numerical formulation

The Saint-Venant equation is not amenable to analytical solution except for a few special cases. They are partial differential equation that, in general, must be solved using numerical methods. To approximate Saint-Venant equation many numerical schemes have been developed. Lax diffusive scheme, which basically uses explicit finite difference method has been implemented for solving the governing equation. The central finite difference method is implemented for spatial derivative and forward finite difference scheme is used for temporal derivative. The final expression using Lax diffusive scheme finite difference scheme is

\[
A_i^{j+1} = (A_i^j)^* - \frac{\Delta t}{2\Delta x} \left( q_{i+1}^j - q_{i-1}^j \right) - f_i^{j+1} \times l
\]
the lateral momentum due to seepage was 

\[ q_i^{+1} = \left( q_i^* \right) + \frac{\Delta t}{2\Delta x} \left( \frac{q_{i+1}^2}{A_{i+1}} - \frac{q_{i-1}^2}{A_{i-1}} \right) - g \times \left( A_i^* \right) \times \Delta x \left( \frac{y_{i+1} - y_{i-1}}{2\Delta x} - s_0 + s_i^* \right) - 0.5 \Delta x \times f_{i+1}^s \times b \times (v_i^{+1})^4 \]  (6)

Where

\[ \left( A_i^* \right) = \frac{A_{i+1}^2 + A_{i-1}^2}{2} \]
\[ (q_i^*) = \frac{q_{i+1}^2 + q_{i-1}^2}{2} \]
\[ s_i^* = \frac{s_{i+1}^* + s_{i-1}^*}{2} \]
\[ f_i^* = \frac{F_i^*}{\Delta t} \]

b is the width of the piedmont zone, v is the velocity of the water. Strelkoff (1970) defined the lateral momentum due to seepage was

\[ m_i = 0.5 q_i^* b \]

For computing infiltration through the recharge zone at any instant of time, length of the moving wetting front is calculated first as it influences the infiltration rate. Total infiltration is then computed by using numerical form of the Green-Ampt equation as given in Eq. (7), Eq. (8)

\[ L_i^{+1} = L_i^* - \Delta t k_s \frac{h_i^* + L_i^* - h_f}{(\theta_s - \theta_i)} L_i^* \]  (7)

\[ F_i^{+1} = F_i^* - \Delta t k_s \frac{h_i^* + L_i^* - h_f}{L_i^*} \]  (8)

Where F is the total infiltration, \( h_0 \) is the initial water surface height, \( L(t) \) is the length of the moving wetting front, \( h_f \) is the dry suction head, normally negative value, \( \theta_s, \theta_i \) are the saturated and initial soil moisture content, \( k_s \) is the saturated hydraulic conductivity.

### 3.1 Stability condition

The stability of the above numerical scheme was checked by the Courant–friedrichs–Lewy (CFL) criterion. The CFL criterion for one dimensional flow is expressed as

\[ \frac{(v + g \sqrt{gh}) \Delta t}{\Delta x} \leq 1 \]  (9)

### 4. Initial condition

A hypothetical river reach of 21 km was considered to have three different slopes in the longitudinal direction. Upper reach of 10 km has a slope of 1:2000, middle reach of length 5 km has a slope of 1:3000 and the lower reach of 6 km has a slope of 1:2500. The cross-sectional dimensions of all the three reaches are considered to be same. The width of the river was considered as 1 km. A steady gradually varied flow profile computed for the initial
discharge is considered as initial condition. Figure 1 represents the initial condition and reach of the hypothetical river.

**Figure 1:** Initial condition

### 4.1 Boundary Condition

Numerical form of the Saint – Venant equation is used in the interior grid points to compute the unsteady flow and velocity. At the boundaries, however, we cannot use this equation, since there is no grid point outside flow domain. Therefore for one dimensional unsteady flow, we need two boundary conditions. One is upstream boundary condition and the other is downstream boundary condition.

### 4.2 Upstream boundary condition

The discharge hydrograph as shown in Figure 2 has been taken as the upstream boundary condition.

**Figure 2:** Upstream boundary condition
4.3 Downstream boundary condition

To calculate the flow parameter at the downstream boundary we have used two equations, positive characteristic equation [Eq. (10)] and Manning’s equation [Eq. (11)]

\[ v_{i-1}^{j} + 2c_{i-1}^{j} = v_{i}^{j+1} + 2c_{i}^{j+1} \]  
(10)

For \( \frac{dx}{dt} = v + c \)

\[ v = \frac{1}{n} y^{2} s_{j}^{\frac{1}{2}} \]  
(11)

Where \( v_{i}^{j} \) is the velocity at \( i^{th} \) grid in space and \( j^{th} \) grid in time. \( c_{i}^{j} \) is the celerity at \( i^{th} \) grid in space and \( j^{th} \) grid in time. Celerity \( c \) is computed by using expression for rectangular channel as \( c = \sqrt{gy} \). Where \( g \) is the acceleration due to gravity and \( y \) is the flow depth.

4.4 Intermediate boundary condition

Once the flow, reaches the piedmont zone, the flow process changes. Therefore an intermediate boundary condition was introduced at the upstream of the piedmont zone. The intermediate boundary is solved by the positive characteristic equation and manning’s equation as discussed earlier in case of downstream boundary condition.

4.5 Result and discussion

The elevation at different time, discharge hydrograph and the depth hydrograph at downstream section are presented in this section. The data used in the model are hypothetical. To assess influence of piedmont zone on the computed flow, the model was first run, without considering the existence of piedmont zone and then by considering the piedmont zone.

4.5.1 Elevations

The elevation of water surface profile without considering the recharge is shown in the Figure 3. Figure 4 represents the water surface elevation considering the infiltration from the river bed. Comparison of these two figures has revealed that the water surface elevation at the downstream of the recharge zone decreases as significant amount of water infiltrates in to the ground through the recharge zone. Sensitivity analysis of hydraulic conductivity ‘k’ has shown that the amount of infiltration depends upon the permeability characteristic of the piedmont zone. Thus, it is necessary to estimate hydraulic conductivity with utmost precision. Figure 5 shows the elevation of the water surface considering different values of hydraulic conductivity for the recharge zone.
Figure 3: Water surface elevation without considering piedmont zone

Figure 4: Water surface elevation considering piedmont zone

Figure 5: Water surface elevations after 100 min
4.5.2 Discharge and depth hydrograph

The discharge and depth hydrographs computed with and without recharge zone are shown for a section at 15 km in Figure 6, Figure 8 respectively. From these figures it has been seen that due to presence of recharge zone in the river, the discharge and depth attenuates significantly at downstream. Reduction in peak was computed to be in the order of 1000 m$^3$/s which is approximately 4% of the peak flow without recharge zone. Similarly depth attenuation was computed as 11%. From this it is cleared that due to presence of recharge zone in a river a significant amount of water moves as subsurface flow. Again Figure 7, Figure 9 shows the variation in discharge and depth hydrograph for different values of hydraulic conductivity and ‘k’ value has been found to be quite sensitive in computation of discharge and depth.

![Discharge hydrograph with and without recharge zone](image1)

**Figure 6** Discharge hydrograph with and without recharge zone

![Discharge hydrograph for different values of hydraulic conductivity](image2)

**Figure 7**: Discharge hydrograph for different values of hydraulic conductivity
5. Conclusions

A mathematical model for computing flood propagation in a river having piedmont zone is developed. Study has revealed importance of considering effect of piedmont zone in flood routing. As seen in this study error induced in the computation of depth hydrograph and discharge hydrograph can be as high as 11% and 4% respectively if the effect of piedmont zone is neglected in the computation of unsteady flow. It has also been seen that the ground water recharge depends upon the geological condition of the river. The value of hydraulic conductivity of the piedmont zone has a great influence on the ground water recharge. Results are found to be quite sensitive to the hydraulic conductivity (k). Therefore ‘k’ value should be determined with high precision, preferably in the field to ensure reliable result.
6. References


