Genetic algorithm based optimum design of prestressed concrete beam

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doi:10.6088/ijcser.2 201203013059

ABSTRACT

Design of prestressed concrete (PSC) beam involves many design variables. Simultaneously handling many design variables computationally expensive and in most cases finds relative optima. An efficient optimization technique, Genetic Algorithm (GA) is well suited for solving such problems and they can find global optimum solution. In the present paper six design variables are considered for optimizing the cost of simply supported prestressed concrete beam with all practical constraints. Since absolute cost of materials changes from time to time, total cost of the beam is expressed in terms of cost ratio. Parametric study has been done to observe the effect of size of population, number of generation, intensity of live load, cable profile and various practical beam lengths. Finally data base is prepared for various practical span lengths.

Keyword: Genetic algorithm, cost ratio, prestressed concrete beam.

Nomenclature

P = Prestressed force
e = eccentricity
B = beam width
D = depth of beam
Dc = diameter of the cable
Aco = Area of the concrete
ρco = density of concrete
Crco = cost ratio of the concrete
f₁ = stress in top fiber
f₁t = permissible stress in tension at transfer
Mg = moment due to dead load
Zₜ = section modulus at top
f₂ = stresses at bottom
fw = permissible tensile stress in concrete under working load
η = prestress loss ratio
Mq = moment due to live load
Zb = section modulus at bottom
W = UDL on beam
E = young's modulus
I = moment of inertia
fₘᵢₜ = minimum required prestress at the bottom fiber
fₛₜₜ = maximum tensile prestress at the top fiber
1. Introduction

Prestressed concrete is a method for overcoming concrete's natural weakness of concrete in tension. It is used to produce beams, floors or bridges with a longer span. Prestressing tendons are used to provide a clamping load which produces a compressive stress that nullifies the tensile stress induced in the beam due to a bending load. Traditional reinforced concrete structure is based on the use of steel reinforcement bars, rebar, inside concrete to counter the bending stress.

Lefebvre (2002) has stressed on shrinkage and creep effect on prestressed concrete structures. It is concluded that final stress depends on mix of differential shrinkage, prestress loss, secondary moments due to prestress and the state of stresses in the section. Laskar et.al, (2008) predicted the structural behavior of prestressed concrete wall-related structures subjected to reversed cyclic loads. Design of prestressed concrete beams and girders has been explained by Krishnaraju (2010).

Large number of papers has been published on structural optimization in recent years. PSC beam design involves large number of design variables and many practical constraints: Hence optimum design of a PSC beam is challenging to the designers. It is computationally expensive to design manually. Hence optimization techniques are more suitable in such case. Thakkar and Rao (1974) used linear programming for optimal design of prestressed concrete pipes. Kirsch (1997) used two-level optimization of prestressed structures. Erbatur et.al, (1991) studied optimization and sensitivity of prestressed concrete beams, objective being both minimum weight and minimum cost formulations for simply supported beam having different sections. Quiroga et.al, (1991) applied optimization techniques in prestressed concrete bridge decks for simply supported beam, continuous beam, portal frames. The cost function is the most appealing design criterion compared to volume or weight Al-Gahtani et.al, (1993).

Barat et.al, (2003) carried out single objective using reliability. The general approach to the single objective reliability- based optimum (SORBO) design of prestressed concrete beams (PCB) is presented. Kirsch (1972) used optimization of prestressed beams for indeterminate uniform section. Optimum values of prestressing force, tendon configuration and cross sectional dimensions are determined subjected constraints on design variables and stresses.


Genetic Algorithm method, which is one of the artificial intelligence methods was the first proposed by Goldberg (1989). One of the first applications of genetic algorithm to civil engineering problems was done by Rajeev and Krishnamoorty (1992), and the method was applied three-bar truss system problem, in detail. GA is a robust tool to find global minima and can also handle mixed design variables. Lute and Upadhyay (2008) applied GA for

In the study made by Ramaswamy and Rajasekaran (1996), various truss configurations are designed using the expert system developed and the same configurations are optimized using Genetic Algorithm. Turgut et.all, (1997) discussed the optimization of a simply supported reinforced concrete beam by this method. Daloglu and Armutcu (1997) investigated the optimum design problem of truss systems via GA. In this problem, constraints are tension, displacement and stability. In the studies of Wei et.all, (1997) an algorithm was formed for time, cost, work optimization based on the principles of GA. In the study of Friswell, (1998) GA is applied to the problem of damage detection using vibration data. The Genetic algorithm is used to optimize the discrete damage location variables. GA conducts a global search to identify the optimal reinforcement bar size and detailing arrangement (Rafiq and Southcombe, 1998). In the study of Saka (1998), GA was presented for the optimum design of grillage systems to decide the cross-sectional properties of members from a standard set of universal beam sections. Pezeshk et.all, (2000) presented a GA-based optimization procedure for the design of 2D and geometrical nonlinear steel-framed structures.

In Civil Engineering Structures, majorly cost is the objective of optimization of the structure. In most cases, the material cost is included in objective function. In some cases, construction cost is also included in objective function. Han and Vautrin (2003) presented a multi-constraint optimization methodology for the design of composite laminated plates using GA. Optimum design of reinforced concrete is carried out by Leps and Sejnoha (2003) with the design objective of minimizing the total cost of a structure. Sahab et.all, (2005) discussed cost optimization of reinforced concrete slab buildings according to the British Code of Practice (BS8110). The objective function is the total cost of the building components. The paper presented by Govindaraj and Ramaswamy (2005) includes the application of genetic algorithm for the optimum design of reinforced concrete continuous beam. The paper prepared by Castilho et.all, (2007) describes the use of a modified GA as an optimization method in structural engineering for minimizing the production costs of slabs using precast prestressed concrete joints. Saini (2007) worked on least-cost design of singly and doubly reinforced beams with uniformly distributed and concentrated load. In the present work cost of materials is taken as objective function in terms of cost ratio and GA is adopted as an optimization tool. All practical constraints are considered in the optimum design of PSC beam. The motivation of this work is to carry out optimum design of prestressed concrete beam with multiple design variables. It is also to explore the possibility of application of Genetic algorithms for optimum design of prestressed concrete structures. The effect of cable profile in the optimum design is attempted. For example, prestressed concrete box girder involves large number and mixed type of variables. Then, handling such variables becomes computationally inefficient without the application of optimization techniques. Hence application of genetic algorithm, in the prestressed concrete structure, is attempted. For small lengths of beam, the cable profiles do not influence the optimum design. The optimum size of population for the given problem is identified. Effect of practical constraints on optimum design has significant variation for long span beams.

2. Problem formulation

The objective of the optimization is to minimize the cost of the prestressed concrete beam while satisfying strength and serviceability constraints. The selected objective function is the total material cost, which consists of the cost of the concrete and prestressing cable. The design variables, objective functions, and practical constraints are explained below.
2.1 Design variable

In this present Optimization model, the parameters related with design, configuration and cross-section of the prestressed beam are taken as design variables.

1. Prestressing force (P)
2. Eccentricity (e)
3. Width of the beam (B)
4. Depth of the beam (D)
5. Profile of the cable (straight and parabola)
6. Diameter of the cable (Dc)

2.2 Objective function

The fitness of an individual is an indicator of how well an individual is suited to its current environment. All the constraints are evaluated in fitness function. Based on the violation of constraint the objective function is penalized. Mathematically, for a set of design variables 'x' the modified objective function can be represented by Equation 1

\[ f(x) = LA_{ca} \rho_{ca} C_{rco} + A_{ca} L_{c} \rho_{ca} C_{rca} \] ----- 1

2.3 Design constraints

One of the important factors that influence the performance of GA is constraint. It is the functional relationship among the design variable and other parameters satisfying certain conditions. The practical design constraints considered for the present work are listed in equation (2) to (7).

Stresses in top (ft) = \( f_{tt} = \frac{N_{t}}{Z_{t}} \) ----- 2

Stresses in bottom (fb) = \( f_{bb} = \frac{N_{t} + N_{p}}{Z_{b}} \) ----- 3

Deflection (\( \delta \)) = \( \frac{3pl^4}{48E_2I} \) ----- 4

Shear strength (Vcw) = \( 0.67bh(f_{t}^{2} + 0.8f_{cp}f_{t})^{1/2} + \eta p \sin \theta \) ----- 5

Eccentricity (e) = \( \frac{Z_{p} - Z_{b} + \left( A_{1} - A_{2}\right)}{A_{1} - A_{2}} \) ----- 6

Minimum prestressing force (P) = \( \frac{A_{1}f_{cp}e + A_{2}f_u(1-e)}{Z_{2} - Z_{1}} \) ----- 7

3. Modeling of beam

In this present problem the prestressed simply supported concrete beam having uniformly distributed live load and dead load are the input parameters. Two cable profiles (straight with constant eccentricity below neutral axis and parabolic profile with maximum eccentricity at mid span and zero eccentricity at both supports) are considered for pre stressing of the PSC beam as shown in Figure 1. From these input parameters bending moment, shear force and deflection are calculated using MATLAB program. This paper deals optimization of prestressed concrete beam using GA. The working principle of GA is explained with the help
of a flow chart shown in Figure 2. Cost of any material is not constant factor at all times. But the ratio of cost of cable to cost of concrete is going to remain almost constant. Hence, instead of total cost of beam to be objective function, cost ratio is considered as objective function in the present problem. The cost ratio for cable to concrete is taken as 8. Which means ratio of cost of cable to cost of concrete for the same volume is 8.

![Figure 1: PSC beam model](image)

The analytical expressions involved in optimum design by Genetic algorithms are as follows.

For the design variables, x, the objective function is calculated using equation -1

According to the violation of constraint the objective function is modified. The objective function is called modified objective function.

Modified objective function \( \phi (x) = f(x) [1 + k \sum_{i=1}^{n} \left( \frac{c_i}{c_{\text{opt}}} - 1 \right) ] \)  

When the algorithm converges, the optimum solution is noted.

The limits of the design variable are given as

1000≤ Prestressed force (kN) ≥ 2000
0.02≤ eccentricity(m) ≥ 0.2
0.2≤ width of the beam(m) ≥ 0.3
0.3≤ depth of the beam(m) ≥ 0.7
1≤ cable profile ≥2
3≤ diameter of prestressed cable (mm) ≥ 12

Step by step procedure to find design of prestressed rectangular concrete beam

1. Calculate the moments due to live and dead loads.

2. Range of stress at bottom fiber \( f_{br} = \frac{M_c}{W_{br}} - f_{cru} \)
3. Minimum section modulus $= \frac{M_2 + (s-2) \cdot M_{\text{c}}}{{f}_{\text{c}}}^{2/3}$  

4. find out the dimension of beam like width and depth

5. Again find the moments due to calculated dimensions

6. Stress at sup $f_{\text{sup}} = \frac{M_2}{s} - \frac{M_{\text{c}}}{s^2}$  

7. Stress at inferior $= \frac{f_{\text{in}}}{q} + \frac{M_2 - M_{\text{c}}}{q \cdot s^2}$  

8. Minimum prestressing force using equation -7

9. Eccentricity using equation -6

![Flow chart for GA optimization](image)

**Figure 2:** Flow chart for GA optimization

### 3.1 Validation

The genetic algorithm program developed by the author, is validated using standard 3 bar truss (Venkat and Upadhyay, 2009) shown in Figure.3. The developed GA program is also applied in the optimum design of cable stayed bridges (Venkat, 2010). The optimum solution is for the standard three bar truss is given in the table 1.
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![Figure 3: Standard three bar truss](image)

Table 1: Standard 3-bar truss optimum solution

<table>
<thead>
<tr>
<th>size</th>
<th>A1 (mm²)</th>
<th>A2 (mm²)</th>
<th>f (x) (N)</th>
<th>σ1 (MPa)</th>
<th>σ2 (MPa)</th>
<th>σ3 (MPa)</th>
<th>def (mm)</th>
<th>φ (x) (N)</th>
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<td>141.78</td>
<td>-1.41</td>
<td>146.67</td>
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</table>

4. Results and discussions

4.1 Effect of size of population on cost ratio

Appropriate population size in GA is needed to avoid trapping in local minima. For 15m beam length, optimization has been carried out for 8 different population sizes. Population size, 8, 10, 12, 14, 16, 18, 20 and 22 are studied. The convergence study (Figure 4) indicates that when the population size is approximately equal to the chromosome length, 20 the solution is optimum. With this population size, one beam design problem is solved at different times and minimum deviation from mean optimum cost is found. This indicates that the selected size of population is appropriate for current problem.

![Figure 4: Size of population Vs optimum cost](image)
4.2 Effect of beam length on optimum cost

Beam lengths varying from 5m to 15m are taken for studies. For each beam length optimum cost is obtained. From Figure 5, it is clear that as beam length increases linearly, the modified cost increases non-linearly. The percentage variation in optimum cost from 14m and 15m beam length is 21.7%.

![Figure 5: Beam length Vs optimum cost](image)

4.3 Effect of live load on optimum cost

In the present problem live loads of uniformly distributed load varying from 20kN/m to 60kN/m are taken. The effect of these live loads on optimum cost has been studied. It is observed from the Figure 6 that the optimum cost linearly increasing with the live load. So, the percentage difference in optimum cost from 50kN to 60kN of live load is 16.8%.

![Figure 6: Live load Vs optimum cost](image)
4.4 Effect of profile of the cable on optimum cost

In some situations practically, particular type of cable profile has to be adopted unavoidably. In such situations, the variable has to be restricted while optimizing with GA. By constraining cable profile variables the variation of modified cost from its optimum cost is explained in Figure 7. The percentage increases in the modified cost by taking the cable profile to be restricted to parabolic is 4.22% for a beam length of 15m. The study reveals that up to beam length of 13m there is no effect of restraining the cable profile but for 14m and 15 m beam lengths parabolic cable profile gives higher optimum cost as compare to straight cable profile.

![Cable Profile Vs Optimum Cost](image)

**Figure 7: Cable profile Vs optimum cost**

In this paper, beam lengths varying from 5 m to 15 m are considered. Optimum cost is tabulated in Table 1 along with design parameters. This data base can be ready made use for designer for working out preliminary cost of PSC beams for various practical span lengths.

**Table 2: Optimum values of design variables for various beam lengths**

<table>
<thead>
<tr>
<th>Length (m)</th>
<th>Optimum Cost</th>
<th>Prestress force (KN)</th>
<th>Eccentricity (m)</th>
<th>Beam width (m)</th>
<th>Beam depth (m)</th>
<th>Cable profile</th>
<th>Dia. of cable (mm)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>903.7</td>
<td>1916.3</td>
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<td>0.254</td>
<td>0.687</td>
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<td>9.4</td>
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<td>3</td>
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<tr>
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<td>0.039</td>
<td>0.209</td>
<td>0.694</td>
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<td>1369.6</td>
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<td>0.695</td>
<td>1</td>
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<tr>
<td>9</td>
<td>5184.8</td>
<td>1773.7</td>
<td>0.164</td>
<td>0.229</td>
<td>0.699</td>
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<td>5.5</td>
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<tr>
<td>10</td>
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<tr>
<td>14</td>
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<td>0.109</td>
<td>0.206</td>
<td>0.672</td>
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</table>
5. Summary and conclusion

Genetic algorithm is robust tool for optimization of structures. In the present paper GA is used to optimize a simply supported prestressed concrete beam with live load and dead loads. Considering all practical constraints and most crucial design variables, the optimum studies are carried out using GA. The following conclusions are drawn from present work:

1. GA can handle more number of variables easily the developed GA program is general to accommodate discrete and continuous variables.
2. Effect of size of population in the present study reveals that appropriate size of population is 20.
3. The percentage difference in optimum cost from 50kN/m to 60kN/m of live load is 16.8%
4. Up to beam length of 13m there is no effect of restraining the cable profile but for 14m and 15 m beam lengths parabolic cable profile gives higher optimum cost as compare to straight cable profile. For beam length of 15 m, the percentage increase in Optimum cost is 4.22% for the parabolic cable profile as compare to straight cable profile.

6. References


