Structural Modelling of Stability of Plane Sway Frames
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ABSTRACT

This paper presents the results of testing of experimental models with varying beam to column stiffness ratios with the aim of establishing their relationship on the stability of selected rigid frames. Test frames with beam to column stiffness ratios of 0.5, 0.67, 1.0, 1.33 and 2.0 were fabricated, and subjected to compressive loading in the laboratory. The results obtained from the tests after application of appropriate scale factors are compared to the prototype critical loads obtained from the sidesway stability function approach by Horne and Merchant.

Results from the analysis show accuracy as high as 98.5% when compared to the results obtained from numerical methods.

The results of the tests further emphasise the criticality of scale factors when using modelling approach for solution of structural stability problems.

Keywords: Frames, Stability, Stiffness, Modelling, Critical Load.

1. Introduction

Frames can be classified as one of the most important naturally occurring and manmade support systems in existence and their applications have continued impact tremendously on the economics of physical infrastructure development worldwide.

The efficiency of a frame no matter its configuration, is characterized by its load carrying capacity of its columns, and this capacity is defined by the critical load that will cause the frame to become unstable. Researchers have continued to investigate means of accurately determining these critical loads in order to optimise the use of frames.

Numerous approaches exist for the evaluation of capacities of frames, popular amongst which are analytical methods. However with increasing size, geometric complexity and accompanying variation in boundary conditions, the concept of structural modelling as a means of solution to frame stability problems can be used as an effective alternative approach.

Structural modelling is defined as the assembly of structural element(s) built to a reduced scale which is to be tested, and for which the laws of similitude must be employed to interpret test results. This definition highlights the model-prototype similitude relationship which is catered for by the scale factor employed during modelling, and subsequently in interpretation.

The application of structural models in Structural dates back several centuries. An extensive review of their development is a subject of such publications as discussed by Sabins et al.
Modelling is applied mainly as a solution to investigation of complex structural systems, employing concepts as similitude dimensional analysis.

2. Structural Modelling of Plane Sway Frame

Destructive testing remains one of the most accurate means of verifying load carrying capacity of structures. Logically, the closer a model is to its actual size, the more accurate the test data obtained from it, if the loading on the structure is accurately simulated. Life size modelling is very challenging in terms of cost of fabrication, loading equipment, instrumentation and testing. In most cases, prototype testing is almost impossible from the economic standpoint.

The relevance of structural models in the study, design and analysis of structural systems is justified by the great savings resulting from significant reduction in loading and size of models in comparison with those required in the corresponding prototype tests. Furthermore, the application of structural models enables a wider participation in structural research in those areas where sophisticated laboratory facilities and loading systems are not available.

The issues of size effects have been raised in various works. However, the authors are in the agreement with the position that size effects are an intrinsic characteristic of models since the atomic and molecular dimensions can never be modelled to a given dimensional scale factor. The reliability of structural model is further substantiated by the fact that carefully crafted test models have been employed to reproduce, with reasonable high accuracy, both qualitatively and quantitatively the behaviour of structural systems.

Structural modelling has also received serious attention in the study of Ephraim et al, who investigated the load carrying capacity of sandcrete masonry wall on a series of 1:10 structural model. Meanwhile, Uzoewhulu has recently mobilized the modelling technique in a successful study of the lateral resistance of reinforced concrete frames, in-filled with cement-stabilized laterite blocks under racky loading. It is interesting to observe that the results of these investigations show a close agreement of prototype and model behaviour up to the ultimate states of failure.

While analytical approaches to stability of plane sway frames have received significant attention in the researches of Merchant, Horne et al, Hoenderkamp [6] and Orumu . Technical literature also reveals a considerable number of experimental works with models of sway frames. A great number of these researches are associated with investigation of stability of sway frames in reinforced and pre-stressed concrete materials, as shown in the modelling approach by Fischer et al ; they however do not apply the principles of similitude relationship to their work, or application of similitude principles for extrapolation of laboratory data for application to life structures through the use of scale factors.

This research is an effort by the authors to extend the structural modelling process to the stability studies of frames using the operational and predictive requirements evolved from similitude mechanics.

3. Methodology

The aim of this research is to produce direct models to represent prototype frames with dimensional properties in three categories. The models are to be tested to failure, and the
results interpreted using the principles of similitude to verify the scaled up result with results obtained for the prototype by mathematical means.

The study is thus presented in two parts, namely: dimensional analysis and laboratory experimental tests. The details of these are covered in the relevant subsections, which follow.

3.1 Dimensional analysis

The governing equation of the critical load for the one storey plane frame is assumed to be of the form

\[ F(P, E, L, H, A, I) = 0 \]

equation 1

Where the assumed independent variables have the following definitions:

P- Critical Load
L- Span of Frame
H- Height of Frame
E- Modulus of Elasticity of frame material
A- Cross sectional Area of column
I- Moment of Inertia of column.

Assuming the modulus of elasticity E and the representative length L as dimensionally independent variables, equation 1 can be recast into an explicit form with dimensionless ratios:

\[ G\left(\frac{PL^2}{E}, \frac{k_b}{k_c}\right) = 0 \quad \text{equation 2} \]

\[ \frac{El^2}{P} = \phi\left(\frac{k_b}{k_c}\right) \quad \text{equation 3} \]

\[ P = \frac{1}{El^2} \phi\left(\frac{k_b}{k_c}\right) \quad \text{equation 4} \]

Since the constant P must be the same for model and prototype, the predictive condition is obtained by forcing the dimensionless ratios:

\[ \frac{P_p}{P_m} = \frac{E_m l_m^2}{E_p l_p^2} \phi_p\left(\frac{k_b}{k_c}\right) \quad \text{equation 5} \]

Thus, the predictive scale factor for the critical load \( S_p = S_L^{2}S_E P_m \)

Assuming the same material in model and prototype, i.e. \( S_E = 1 \), the derived similitude conditions for loads, material properties and geometry are given in Table 1.

**Table 1:** Prototype and Model Properties for sample P1

<table>
<thead>
<tr>
<th>S/no</th>
<th>Property</th>
<th>Dimension</th>
<th>Scale Factor</th>
<th>Prototype</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>Loading</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Critical Load</td>
<td>F</td>
<td>( S_L^{2} )</td>
<td>( P_{CR(P1)} )</td>
<td>( P_{CR(model)} )</td>
</tr>
<tr>
<td></td>
<td>Stress</td>
<td>FL^{-2}</td>
<td>1</td>
<td>460 kN/mm(^2)</td>
<td>460 kN/mm(^2)</td>
</tr>
<tr>
<td>2.0</td>
<td>Material Property</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Modulus of Elasticity</td>
<td>FL^{-2}</td>
<td>1</td>
<td>205 kN/mm(^2)</td>
<td>205 kN/mm(^2)</td>
</tr>
<tr>
<td></td>
<td>Poissons Ratio</td>
<td>-</td>
<td>1</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>Density</td>
<td>FL^{-3}</td>
<td>1</td>
<td>76.82 kN/mm(^3)</td>
<td>76.82 kN/mm(^3)</td>
</tr>
</tbody>
</table>
3.2 Experimental materials and procedure

The experimental aspects of the research were conducted in the Structures laboratory of the Rivers State University of Science and Technology, Port-Harcourt, Nigeria.

3.2.1 Materials

The materials employed in the fabrication of the test modes is high yield carbon steel reinforcement bars, and its properties are summarised in the Table 1 above.

3.2.2 Model properties

The plane frames were fabricated as two storey plane frames. The frames are classified according to their beam to column stiffness ratios in the value: 0.5, 0.67, 1.0, 1.33 and 2.0.

The columns are fabricated as continuous members, with the beams framing into the columns at the first and second floors, connected via full fillet welds at the beam column interface.

3.2.3 Experimental procedure

A total of five plane frame samples were fabricated to the specifications outlined in sections 3.2.1, and 3.2.2. The objective of the test is to load the frame to failure in a bid to determine experimentally the critical load of the frame.

Axial load application from the loading frame to the test frame is controlled in such a way that each of the frame columns receives approximately the same amount of load. This loading was achieved via the use of the fabricated load transfer frame illustrated in Figure 2.

<table>
<thead>
<tr>
<th>Column</th>
<th>Beams</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>S_L</td>
<td>S_L</td>
</tr>
<tr>
<td>3000</td>
<td>300</td>
</tr>
<tr>
<td>L_1</td>
<td>L_1</td>
</tr>
<tr>
<td>S_L</td>
<td>S_L</td>
</tr>
<tr>
<td>3000</td>
<td>300</td>
</tr>
<tr>
<td>L_2</td>
<td>L_2</td>
</tr>
<tr>
<td>S_L</td>
<td>S_L</td>
</tr>
<tr>
<td>2250</td>
<td>225</td>
</tr>
<tr>
<td>L_3</td>
<td>L_3</td>
</tr>
<tr>
<td>S_L</td>
<td>S_L</td>
</tr>
<tr>
<td>1500</td>
<td>150</td>
</tr>
<tr>
<td>L_4</td>
<td>L_4</td>
</tr>
<tr>
<td>S_L</td>
<td>S_L</td>
</tr>
<tr>
<td>1125</td>
<td>112.5</td>
</tr>
<tr>
<td>L_5</td>
<td>L_5</td>
</tr>
<tr>
<td>S_L</td>
<td>S_L</td>
</tr>
<tr>
<td>750</td>
<td>75</td>
</tr>
</tbody>
</table>
3.2.4 Experimental set-up and instrumentation

The experimental set-up is as illustrated in Figure 3, which shows a schematic of the set-up. Specimens were placed on the loading frame platform. Spirit level and shim plates were used to ensure plumbness of the frames as they were being placed on the frame. When plumbness was achieved, the test frames were firmly secured to the loading frame base by bolting through the slots provided in the test frame supports.

![Figure 3: Schematic of Test Set-up](image)

The load transfer mechanism was then placed on the frame ensuring that each of its four legs placed directly on the top of each of the frames’ column. The loading frame was also checked for plumbness, using a spirit level and shim plates.

After verification of plumbness of the test frame and load transfer mechanism, the test frame was loaded with increments of 5kN until failure.

The instrumentation employed for the tests was mainly the load gauge of ELE make, appropriately calibrated prior to use. Plate 1 shows the test set-up and the testing process of a typical frame.

![Plate 1: Test Set-up](image)
4. Discussion of results

The results obtained from the prototype frames and compared to the predicted results are presented in this section.

Table 2 below shows a summary of the theoretical results obtained for the prototype (P1, P2, P3….) under consideration, the equivalent test results obtained from the direct model and the predicted results obtained using the scale factors for various beam to column stiffness ratio for both prototype and model k_b/k_c. The model critical load P1 were obtained from the laboratory exercise; the prototype critical load were calculate using the Horne and Merchant method; and the predicted critical load expressed as the model critical load multiplied by the scale factor obtained from the prototype-model similitude characteristics.

<table>
<thead>
<tr>
<th>S/no.</th>
<th>SAMPLE</th>
<th>k_b/k_c</th>
<th>P1 (Prototype)</th>
<th>P1 (Model)</th>
<th>P1 (Predicted)</th>
<th>P2 (Prototype)</th>
<th>P2 (Model)</th>
<th>P2 (Predicted)</th>
<th>P3 (Prototype)</th>
<th>P3 (Model)</th>
<th>P3 (Predicted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-1</td>
<td>0.50</td>
<td>3089.86</td>
<td>30.00</td>
<td>3000.00</td>
<td>6922.19</td>
<td>6750.00</td>
<td>12359.45</td>
<td>12000.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2-1</td>
<td>0.67</td>
<td>4060.96</td>
<td>40.00</td>
<td>4000.00</td>
<td>9137.16</td>
<td>9000.00</td>
<td>16243.84</td>
<td>16000.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3-1</td>
<td>1.00</td>
<td>4855.50</td>
<td>45.40</td>
<td>4540.00</td>
<td>10924.87</td>
<td>10215.00</td>
<td>19421.99</td>
<td>18160.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4-1</td>
<td>1.33</td>
<td>5032.06</td>
<td>49.00</td>
<td>4900.00</td>
<td>11322.14</td>
<td>11025.00</td>
<td>20128.24</td>
<td>19600.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5-1</td>
<td>2.00</td>
<td>6179.72</td>
<td>60.30</td>
<td>6030.00</td>
<td>13904.38</td>
<td>13567.50</td>
<td>24718.89</td>
<td>24120.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Results obtained are typical of direct modelling techniques show that as the beam to column stiffness ratio increases there is a corresponding increase in the critical loads for both the model and the prototype, and the predicted critical loads.

![Figure 4: Graph of P_cr vs. k_b/k_c highlighting the variance in results between predicted critical loads and the prototype critical loads for frames under investigation.](image)

The graphs presented in Figure 4 show plots of the predicted critical loads and prototype results obtained by numerical means. The graph shows the high correlation (98.5% to 93.5%)
of when compared, exhibiting variance of only minimum of 1.5% to maximum of 6.5%. The results highlight how scale factors, which satisfy similitude requirements, affect accuracy of predicted results. The scale factor for the elastic modulus is maintained as unity, representing the similarity in material types between the model and the prototype (this is typical of direct models), while the size factor is modified accordingly according to the prototype to model scale ratio.

The high degree of accuracy of results obtained from modelling, compared to the results obtained by analytical means clearly gives credence to the importance of satisfying similitude requirements when using modelling as a basis for any research for structural purposes. Many laboratory structural investigations used for investigative, verification and validation purposes reach their conclusions without checking to satisfy similitude requirements.

The subject of modelling is subject worthy of detailed attention in design codes, especially when design engineer have to tackle design of complex structures. While section 19.3.3 of ACI 318 permits the use of modelling in shelled structures, the extent of possible range of application of modelling techniques even to simpler problems is however underscored.

Importance of modelling application to structural design and research can be of great economic benefit and advantage to developing countries where reduced scale models can be used at a reduced cost of fabrication, load requirement and instrumentation plus boosting participation in research and development.

5. Conclusion

On the basis of the analysis and discussion of results obtained in the above experimental work, the following conclusions were arrived at.

1. Direct structural model is highly applicable in the study of stability of two storey plane frames.
2. The model yields a near linear relationship between the critical load on frame and its beam to column stiffness ratio. This is an agreement with established trend.
3. There is a close agreement, about 98%, between loads predicted by the model study and those obtained analytically, both for the experimental frame as well as the theoretically simulated frames considered in the study.
4. The importance of modelling techniques can be directly employed in design especially in cases where there are changes in geometric characteristics of the elements under consideration.

6. References


12. Alami, Z.X and Ferguson, P.M., (1963), Accuracy of Models Used in Research on Reinforced Concrete, Proceedings of the American Concrete Institute, 60(11), pp 1643-1663