Numerical modeling of strengthened concrete beams
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ABSTRACT

There are many cases where concrete structures need to be strengthened. Fiber reinforced polymer (FRP) composites, due to their high strength and corrosion resistance properties are widely applied in these cases.

This paper presents a non-linear finite element analysis of reinforced concrete beam strengthened with externally bonded FRP sheets under increasing flexural loads. The different materials models, for concrete, steel reinforcement and fibre reinforced polymers are taken into account. Elastic-plastic model with Drucker-Parger failure criterion and Rankine failure criterion are used to determine the yield surface and the fracture limit of respectively compressive and tensile concrete. Smeared model is used to represent the cracked concrete. Three-dimensional elements for concrete are used. The steel and FRP reinforcement carries only axial forces and considered as unidirectional materials. From the analysis, it is found that FEM can predict the load-displacement relation and good agreements were obtained when compared to the experimental data.

Keywords: Strengthening, Elasto-Plastic Model Reinforced Concrete Beam, FRP, Finite Elements, Failure Criteria.

1. Introduction

Fiber reinforced polymer (FRP) composites have been widely applied to strengthen concrete structural elements. Due to its high tensile strength, low weight and high durability, (Hollaway and Leeming, 1999). FRP has been successfully used to increase the flexural and shear capacities of RC beams, (Saadatmanesh and Ehsani, 1990). Earlier research has demonstrated that the addition of FRP laminate to reinforced concrete beams can increase stiffness and maximum load and reduce crack widths. (Toutanj et. al., 2006) studied the beams retrofitted with FRP laminates. They showed an increase in the maximum load up to 170 % as compared to control beams. (Kachlakev and McCurry, 2000) showed an increase of 150 % when beams were strengthened in both flexure and shear with FRP laminates. Several research have been conducted by (David et. al., 1998), (Shahawy et. al., 1996), (Khalifa and Nanni, 2002), (Shehata et. al., 2001), (Khalifa et. al., 1999) to quantify the flexural and shear strengthening enhancements offered by the externally bonded FRP laminates. (Ferreira et. al., 2003) concluded that the stiffness increase and the tension cracking are delayed to higher loads, when a beam is strengthened with FRP sheets. (Karunasena et. al., 2002) showed that an externally bonded composite, of either CFRP or GFRP materials, improved the moment capacity of deteriorated concrete beams.

2. Significance of research
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Many researchers have simulated the behavior of retrofitted reinforced structures by using the finite element method. Several different approaches have been considered. Some models use nonlinear elasticity or plasticity models to predict the behavior of reinforced concrete retrofitted by FRP (Karunasena et. al., 2002), (Camata et. al., 2007), (Coronado and Lopez, 2006), (Ebead and Marzouk, 2005), (Lundquist et. al., 2005), (Supavitiriyakit et. al., 2004). However, there is no standard reliable design method for preventing the load displacement or stress strain relations. Since the laboratories tests needs an important material to be conducted, the FEM analysis using non-linear constitute models of cracked concrete and reinforcing bars is proposed to predict the load -deflection responses. This leads to the predicting the load capacity of the member. This is a valuable supplement to the laboratory investigations. It is based on the elastic – plastic -modelling. Material nonlinearity due to non-linear stress strain relations of the constitutive materials, cracking of the concrete, plasticity of the reinforcement and of the compression concrete have been incorporated in finite element procedures to analyse the structural response throughout the loading regime.

3. Modeling of materials

Different material models were used with respect to their ability to describe the behavior of the beam. The compressive concrete is considered to behave as elastic-plastic model. Crushing occurs when all principal stresses are compressive and lie outside the failure surface according to Drucker – Prager failure criterion, subsequently, the elastic modulus is set to zero in all directions, and the element effectively disappears. In the tensile region, the smeared crack model was applied to the concrete elements to incorporate the contribution of partially cracked concrete into the stiffness matrix of materials, since cracked concrete can still carry some tensile stress perpendicular to the crack. The tensile stress of crack surfaces is released gradually (Figure 1), and the element stiffness matrix changes from isotropic to lateral isotropic. After the crack forms at an integration point, both normal and shear stiffness are reduced. The tensile failure of concrete occurs when the tensile stress in the principle direction exceeds the tensile strength of concrete, according to Rankine failure criterion. The stress-strain relation for steel reinforcement was assumed as a bilinear isotropic-hardening model.. Yield of rebar was controlled following the Von Mises yield criterion.

The behaviour of FRP materials is linear elastic to failure. Ultimate elongation strains are considerably higher than steel yielding strains. Failure is sudden and brittle with no load carrying capacity after failure. For FRP, the failure criteria were also based on the stress-strain curve. When the principal strain is greater than the ultimate strain, it is assumed tension failure of the elements.

Figure 1: Tension Stiffening Model
4. Finite element formulation

Based on the above relations, the finite element formulation is used to solve the governing differential equations for the required stress and strain distribution. Eight nodded plane stress elements with 3x3 gauss integration points, and truss elements are adopted to represent, respectively, concrete and steel reinforcement. Due to the symmetry of the geometry and reinforcement, only one half of the beam is taken into account, Figure 2. The stiffness matrix of the composite element is obtained by summation of the individual material components, concrete, steel reinforcement and FRP sheets.

The material behaves elastically as long as a limiting stress state described by the yield surface is not exceeded. The displacements increments \( \{\Delta u\} \) are obtained from the equation: \( [K_T]\{\Delta u\} = \{\Delta r\} \) where \([K_T]\) is the tangent stiffness matrix evaluated at the previous iteration and \(\{\Delta r\}\) is the residual load vector. As long as this relation is linear and the stiffness corresponds to\([K_T]\), equilibrium will be gained by default. Beyond the elastic limit, the problem will be thus nonlinear and requires an iterative solver. The Newton Raphson non-linear method is adopted in which the tangent stiffness matrix is updated at the beginning of each iteration. The iterative cycles are repeated until the convergence is reached.

![Figure 2: Geometry of simply supported beam](image)

<table>
<thead>
<tr>
<th>Properties of used materials</th>
<th>Concrete</th>
<th>Steel reinforcement</th>
<th>FRP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus (MPa)</td>
<td>( E_c = 47000 )</td>
<td>( E_s = 200000 )</td>
<td>( E_{FRP} = 138000 )</td>
</tr>
<tr>
<td>Compression strength (MPa)</td>
<td>( f_c = 41.37 )</td>
<td>( f_{sz} = 350 )</td>
<td>( f_{sz} )</td>
</tr>
<tr>
<td>Tensile strength (MPa)</td>
<td>( f_s = 10.34 )</td>
<td>( f_{sz} = 350 )</td>
<td>( f_{sz} )</td>
</tr>
<tr>
<td>Dimensions (mm)</td>
<td>( b = 203 )</td>
<td>( A_s = 265 \text{ mm}^2 )</td>
<td>( A_{FRP} )</td>
</tr>
</tbody>
</table>

5. Results and discussion

In this section, numerical results on the nonlinear response of a simply-supported RC beam are presented and discussed. The geometrical and mechanical properties of the beam are presented in Figure 2 and Table 1.

Figure 3 displays the present load displacement curves at mid span section for a FRP sheets number range from 4 to 0 (without FRP sheet). These curves show reasonable shapes. From
this Figure, the strengthened beams were stiffer and have more load capacity than that without FRP reinforcement.

The increase in flexural capacity and stiffness is due to the contribution of the FRP sheets in the tensile face of the beam. This sheets exhibit elastic deformation to relatively large strain values before rupture and limit the crack propagation. As a result the FRP contributes by additional capacity and stiffness to the beam. However, no significant increase in flexural capacity was observed in case of one FRP layer (about 7% increase ) compared to the case of a beam without FRP, since only one FRP layer seems to be not effective in the flexural capacity. The increase in the flexural capacities reaches its maximum (27%) with four FRP sheets.

The present results were compared with those of (Hoque, 2006) for the case of the beam without FRP and for the beam with 4 FRP sheets (Figure 4), and acceptable agreement was observed.

![Figure 3: Load - deflection curves for different FRP layers number (Nb FRP)](image)

It can be seen from the Figure 5 that increasing the number of layers of FRP increases the beam load capacity ratio (Pc/Pu) and reduces the beam mid-span deflection ratio (dc/du). These curves are nearly linear. It should be noted that only up to two layers of FRP the effect of the reinforcement becomes visible. The variation of the load capacity and deflection ratios were compared with those of (Hoque, 2006) and acceptable agreement was also observed.
Figure 4: Present results (P) compared with those of (Hoque, 2006) (H) for the beam without FRP (0) and with 4 layers.

Figure 5: Comparison of the present load capacity ratio (P – P) and deflection ratio (P – d) with those of (Hoque, 2006) respectively (H-P and H-d).

5.1 Conclusion

Finite element analysis using non-linear constitutive models of cracked concrete, steel bars and FRP is used to predict the effect of the FRP layers on the load deflection curves of reinforced concrete beams, and is also capable of simulating their behavior under the increasing flexural load.

Good correlation has been observed in comparing different curves with those of (Hoque, 2006). It has been, also, confirmed the load capacity and gain strength of the beams can be achieved by increasing the number of FRP layers. se overturning moment.
6. References


