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Fuzzy Risk Analysis Model for Construction Projects

Pejman Rezakhani
M.Sc. Student, Kyungpook National University, School of Civil and Architectural Engineering, 1370 Sankyuk-dong, Buk-gu, Daegu 702-701, Korea
pejman.rezakhani@yahoo.com

ABSTRACT

Compared with many other industries, the construction industry is subject to more risk due to the unique features of construction activities. Risk management aims at identifying sources of risk and uncertainty, determining their impact, and developing appropriate management response. A systematic process of risk management has been divided into risk classification, risk identification, risk analysis and risk response. The ability of a fuzzy system to explain its reasoning process is shown to have definite applicability within the field of risk analysis. Also fuzzy set theory is highly subjective and related to inexact and vague information which we deal in construction projects. This paper describes the development of a fuzzy risk analysis model to assess the risks associated with construction projects. At the end of paper, the proposed model is used to assess the associated risk with a construction operation based on evaluations of three evaluators. The results derived from this model indicate a systematic and effective way for risk analysis.

Keywords: Construction projects; Fuzzy risk analysis; Risk management; EFWA Algorithm

1. Introduction

Concern with uncertainty seems to grow as s field of knowledge matures. This has been observed in the evolution of physics, operations research, economics, financial management and a number of other fields. (Steven Pender, 2001) The construction industry, perhaps more than most is plagued by risk. Too often this risk is not dealt with satisfactory and the industry has suffered poor performance as result (J.H.M Tah and V.Carr, 2000).

Compared with many other industries, the construction industry is subject to more risk due to the unique features of construction activities, such as long period, complicated processes, abominable environment, financial intensity and dynamic organization structure. (Flanagan and Norman, 1993; Akintoye and MacLeod, 1997; Smith, 2003). Hence taking effective risk management techniques to manage risks associated with variable construction activities has never been more important for the successful delivery of a project.

Risk management maybe described as "a systematic way of looking at areas of risk and consciously determining how each should be treated. It is a management tool that aims at identifying sources of risk and uncertainty, determining their impact, and developing appropriate management response.' (Uher, 2003). A systematic process of risk management has been divided into risk classification, risk identification, risk analysis and risk response. (Berkeley *et.al.* 1991; Flanagan and Norman, 1993).

Risk analysis can be conducted by using the theory of probability which estimates the likelihood and consequence of any given risk. Due to some unknown and vague factors which affects project, probability theory cannot deal with important aspects of project uncertainty and cannot explain some important aspects of observed project management practice (Steven Pender, 2001).

The ability of a fuzzy system to explain its reasoning process is shown to have definite applicability within the field of risk analysis. Also fuzzy set theory (FST) is highly subjective and related to inexact and vague information which we deal in construction projects. This paper applies the fuzzy risk analysis model which has been introduced by E.W.T Ngai, F.K.T Wat, (2005), to construction projects. In this paper we followed the same procedure to build the fuzzy risk analysis method but the definitions and applications are customized for construction projects. The authors in respected paper claim that their fuzzy risk analysis method is applicable in other application areas and as a result of this study this method can easily be applied to construction projects to analyze the risks through fuzzy sets and theory.

2 Literature Review

2.1. Classification of risks associated with construction projects

Risk classification is an important step in the risk assessment process, as it attempts to structure the diverse risks that may affect a project. Many approaches have been suggested in the literature for classifying risks. Chen *et.al.* (2004) proposed 15 risks concern with project cost and divided them into three groups: resource factors, management factors and parent factors. Shen (1997) identified eight major risks accounting for project delay and ranked them based on a questionnaire survey with industry practitioners. Abdou (1996) classified construction risks into three groups, i.e. construction finance, construction time and construction design. Chapman (2001) grouped risks into four subsets: environment, industry, client and project. Shen (2001) categorized them into six groups in accordance with the nature of the risks, i.e. financial, legal, management, market, policy and political. Perry and Hayes (1985) give an extensive list of factors assembled from several sources, and classified in terms of risks retainable by contractors, consultants and clients.
In this study we use the Tah and Carr (2000) approach which uses a hierarchical risk breakdown structure to classify risks according to their original and to the location of their impact in the project. The hierarchical risk breakdown structure (HRBS) allows risks to be separated into those that are related to the management of interval resources and those that are prevalent in the external environment (J.H.M Tah and V.Carr, 2000). External risks are relatively uncontrollable and internal risks are relatively more controllable and vary between projects.

2.2. Reasons for using fuzzy risk analysis for construction projects

PMBOK Guide defines risk as a measure of the probability and consequence of not achieving a defined project goal. Risk has two primary components for a given event:

- A probability of occurrence of that event
- Impact (consequence) of the event occurring

Consequently the risk for each event can be defined as a function of probability and consequence (impact); that is: (PMBOK Guide)
Probability theory cannot deal with important aspects of project uncertainty and cannot explain some important aspects of observed project management practice.
Following are some limitations of probability theory:

1. Probability theory is based on assumption of randomness where as project deal with consciously planned human actions that are generally not random.
2. Uniqueness of project reduces the relevant and reliability of statistical aggregates derived from probability-based analysis.
3. Probability theory assumes future states are known and definable; however uncertainty and ignorance are inevitable on projects.

The laws of probability apply if certain assumptions are met, including: (Steven Pender, 2001)

- Knowledge of probable future states
- Rationality
- Frictionless transactions
- Random events
- Repeatability
- Comparability
- Optimization goal

Project parameters and outcomes have *shades of grey*. Take for example the following statements:

"If the schedule slippage is very small and the design changes are great, then the cost impact will not be insignificant."

This type of statement often occurs in projects and reflects the true imprecision of project process and outcomes. Imprecise statements cannot be interpreted within the framework of probability theory because of its assumption of crisp inputs and outputs.

The theory of fuzzy sets provide a framework and offers a calculus to address these fuzzy statements. (Zadeh L.A., 1965) states the theory of fuzzy sets as follow:

`...provides a natural way of dealing with problems in which the sources of imprecision is the absence of sharply defined criteria of class membership rather than the presence of random variables`

Fuzzy logic is a generalization of the traditional bivalent logic which says that any assertion can be true or false, but not both simultaneously.

The primary reasons for using fuzzy logic risk analysis model are:

- The modeling of vague input is successfully done with the use of membership functions
- The inherent ability of fuzzy logic systems to explain its reasoning ensures that the modeling process is understood and could also be intuitively verified.
- The parallel nature in which rules are activated in a fuzzy system ensures that all factors are considered in a harmonized manner.

- The results of fuzzy systems can naturally be scaled to be comparable with each other, with the use of the scaling membership functions.
- Fuzzy logic`s use of linguistic sets and rules ensures that the terminology of the user interface and modeling structure can be tailored toward the specific environments.

Techniques for risk analysis can be either qualitative or quantitative depending on the information available and the level of detail that is required (Bennett J.C. et al. 1996). Statistical approach is the main idea for quantitative techniques. Some tools for this technique are Monte Carlo Simulation (D.White, 1995), Fault and Event Tree Analysis (Bennett J.C. et al. 1996, D.White, 1995), Sensitivity Analysis (D.White, 1995), Annual Loss Expectancy (R.K.J.R. Rainer, et al. 1991), Risk Exposure (B.W. Boehm, 1989), Failure Mode and Effective Analysis (D.White, 1995) etc.

Qualitative techniques rely more on judgment than on statistical calculations such as Scenario Analysis (R.K.J.R. Rainer, et al. 1991), Fuzzy Set Theory (R.K.J.R. Rainer, et al. 1991). Quantitative techniques can involve significant additional expenses and is only warranted in the rare instance where the assumptions of probability theory apply. Among these techniques, the application of fuzzy set theory to risk analysis seems appropriate; as such analysis is highly subjective and related to inexact and vague information (E.W.T Ngai, F.K.T Wat, 2005). In construction research area one of the applications of fuzzy risk analysis is to outline an approach to the assessment of the construction project risk by linguistic analysis.

2.3. Fuzzy Weighted Average

An operation commonly used in risk and decision analysis is the weighted average operation (C.H. Junag, X.H. Huang, D.J. Elton, 1991) which takes the following form:

$$\overline{W} = \frac{\sum_{i=1}^{n} W_i \times R_i}{\sum_{i=1}^{n} W_i}$$

Where $\overline{W}$ is the weighted average of ratings, R_i is the rating according to criterion $_i$ and W_i is the weight assigned to criterion $_i$. When the terms R_i and W_i are represented by fuzzy sets or fuzzy numbers, the above operation is referred to as a *Fuzzy Weighted Average (FWA)*. Schmucker used the FWA to propose an approximate numerical method known as “fuzzy risk analysis” which has many applications particularly in relation to construction projects. *Improved Fuzzy Weighted Average (IFWA)* suggested by (Liou and Wang T.J. Liou, M.J.J. Wang 1992) because the FWA algorithm requires $o(n^2)$ comparisons and arithmetic operations (E.W.T Ngai, F.K.T Wat, 2005). In this study, the *Efficient Fuzzy Weighted Average Algorithm (EFWA)* which presented by (Lee and Park D.H.Lee, D.Park, 1997) is used. The reason for using this algorithm is because its ability to reduce the number of comparisons and arithmetic operations to $o(n \log_n)$ rather than $o(n^2)$, as is the case with the IFWA (T.J. Liou, M.J.J. Wang, 1992).

3. Construct the Fuzzy Risk Analysis Model

The method for fuzzy risk analysis model consists of five stages (E.W.T Ngai, F.K.T Wat, 2005) as:

- Risk classification
- Natural language representation
- Fuzzy assessment aggregation
- Fuzzy weighted average computation
- Linguistic approximation

3.1. Risk classification

The first step is to conduct risk identification and compile a list of the most significant uncertainty factors and their descriptions. As mentioned earlier, in this study we use Hierarchical Risk Breakdown Structure (HRBS*)* based on Tah *et al.* (1993) approach.
In HRBS the project risks are categorized to internal and external risks. External risks are those which are relatively uncontrollable and due to their nature there is a need for continual scanning and forecasting of these risks as like as economic risks, political risks, etc. internal risks are relatively more controllable and vary between projects. Internal risks are divided to local and global risks. The hierarchical risk breakdown structure is shown in figure 1.

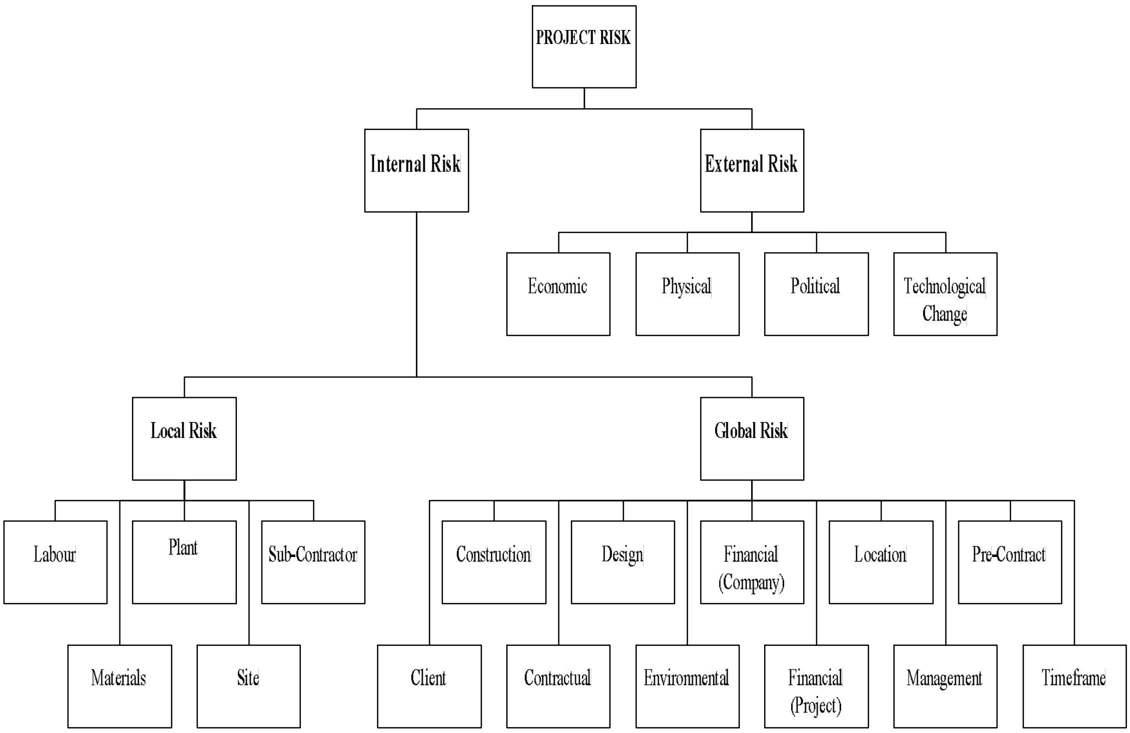


Figure 1: Hierarchical Risk Breakdown Structure (HRBS)

3.2. Natural language representation

The risk assessment process requires an assessment of the probability or likelihood of the risk and impact. The assessment of the level of risk is a complex subject shrouded in uncertainty and vagueness. Risk severity should be considered in terms that are as close as possible to the corporate objectives at the time of assessment. A simple approach that is advocated by some risk experts is to multiply the severity of the consequence by the likelihood of their occurrence, as the likelihood of the occurrence automatically includes the exposure (A.Waring, A.I. Glendon, 1998). Consequently the key attributes of risks and risk factors are likelihood and severity (J.H.M Tah and V.Carr, 2000). Table 1 shows a customizable standard terms for quantifying likelihood and table 2 shows a customizable standard term for severity quantification. In Fuzzy Weighted Average (FWA) we can assign 'likelihood" as the rating factor (R_i) and "severity" as the weighting factor (W_i) that corresponds to the rating factor$_i$.

Table 1: Customizable standard terms for quantifying likelihood

Likelihood	Description
Very very likely	Expected to occur with absolute certainty
Very likely	Expected to occur
Likely	Very likely to occur
Medium	Likely to occur
Unlikely	Unlikely to occur
Very unlikely	Very unlikely to occur
Very very unlikely	Almost no possibility of occuring

Table 2: Customizable standard terms for severity quantification

Severity	Time	Cost	Quality	Safety
Critical	>20% above target	>20% above target	Very poor	Injury
High	10%<target<20%	10%<target<20%	Poor	Safety hazard
Moderate	5%<target<10%	5%<target<10%	Average	Average
Low	1%<target<5%	1%<target<5%	Above average	Above average
Minimal	1%<target	1%<target	OK	OK

3.3. Membership Functions

The difference between traditional set and fuzzy set theory lies in the degree of membership which elements may possess in a set. Traditional set theory dictates that an element is either a member of a set or it is not; its membership values are defined as 1 or 0. In fuzzy set theory this membership value can take any real value from 0 to 1 and this value defines the degree of membership of a given set.

A membership function is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1. The only condition a membership function must really satisfy is that it must very between 0 and 1. There are so many membership functions which can e used. Some of them are Triangular, Trapezoidal, Gaussian, Generalized Bell, Z Curves, etc.

In this study, the membership functions of the linguistic terms are characterized by Triangular fuzzy numbers. Triangular fuzzy numbers are very often use in applications such as fuzzy controllers, managerial decision making, business and finance, social science, etc. (G.Bojadziev, M.Bojadziev, 1997).

A triangular fuzzy number A or simply triangular number with membership function $\mu_A(x)$ is defined on R by

$$A \cong \mu_A(x) = \frac{x - a_1}{a_M - a_1} \quad for\ a_1 \le x \le a_M,$$

$$= \frac{x - a_2}{a_M - a_2} \quad for\ a_M \le x \le a_2, \qquad (1)$$

$$= 0 \quad otherwise,$$

Where $[a_1, a_2]$ is the supporting interval and the point $(a_M, 1)$ is the peak. (Figure 2)

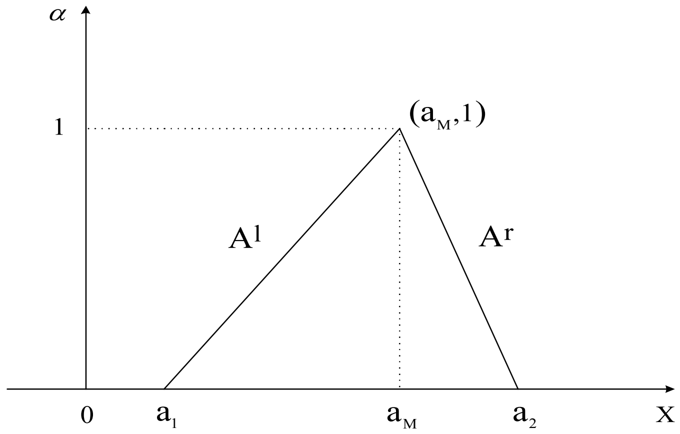


Figure 2: Triangular Fuzzy Number

Often in application the point $a_M \in (a_1, a_2)$ is located at the middle of the supporting interval $(a_M = \frac{a_1 + a_2}{2})$. Then substituting this value into (1) will result the *central triangular fuzzy number* as follow:

$$A \cong \mu_A(x) = 2\frac{x - a_1}{a_2 - a_1} \quad for\ a_1 \le x \le \frac{a_1 + a_2}{2},$$

$$= 2\frac{x - a_2}{a_1 - a_2} \quad for\ \frac{a_1 + a_2}{2} \le x \le a_2, \qquad (2)$$

$$= 0 \quad otherwise,$$

A central triangular number is symmetrical with respect to the axis μ if in (2) $a_1 = -a, a_2 = a$, hence $a_M = 0$. So the triangular number $A = (a_1, a_M, a_2)$ (3) will be denoted by $A = (-a, 0, a)$ and is very suitable to express the word *small* like *small risk.*

The right branch (segment) of $A = (-a, 0, a)$ when $0 \le x \le a$ can be used to describe positive small (PS). We can denote it by $A^r = (0, 0, a)$.

More generally in triangular number (3) $A^l = (a_1, a_M, a_M)$ is suitable to represent positive large (PL) or word with meanings like *high risk.*

Fuzzy set representation for each linguistic term is shown in table 3:

Table 3: Fuzzy set representation for each linguistic terms

Likelihood	Severity	Membership Functions	Triangular Number	Supporting Intervals	
Very Unlikely	Minimal	1-4x	(0,0,0.25)	0≤x≤0.25	P.S.
Unlikely	Low	4x	(0,0.25,0.5)	0≤x≤0.25	
		2(1-2x)		0.25≤x≤0.5	
Medium	Moderate	4x-1	(0.25,0.5,0.75)	0.25≤x≤0.5	
		3-4x		0.5≤x≤0.75	
Likely	High	2x-1	(0.5,0.75,1)	0.5≤x≤0.75	
		4(1-x)		0.75≤x≤1	
Very Likely	Critical	4x-3	(0.75,1,1)	0.75≤x≤1	P.L.

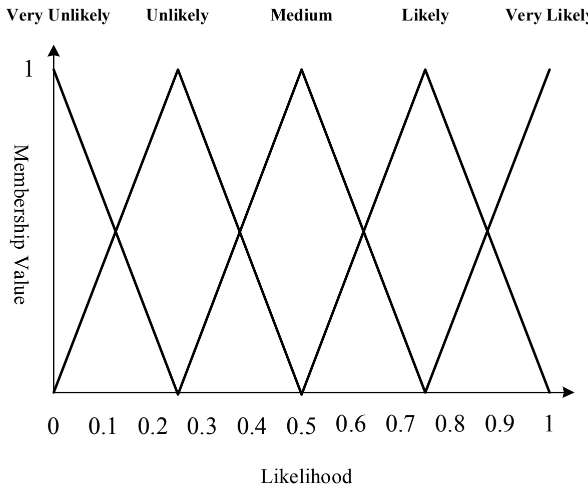


Figure 3: Membership function of likelihood

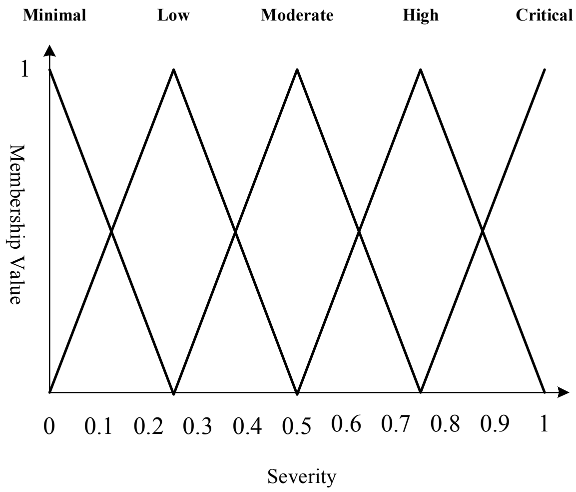


Figure 4: Membership function of severity

3.4. Fuzzy Assessment Aggregation

Expert judgment techniques have the potential for bias in risk identification and risk analysis, as well as in selecting risk response strategies. These biases vary on a case-by-case basis and can affect the probability of occurrence and consequence of occurrence estimates differently. Cognitive factors that can introduce a bias and/or noise term include, but are not limited to:

- Adjustments from an initial value
- Anchoring (biased toward the initial value)
- Availability of post events
- Fit ambiguous evidence into predispositions
- Insensitivity to the problem or risk
- Motivation
- Overconfidence in the reliability of the analysis
- Overconfidence in one`s ability
- Proximity to project
- Relationship with other experts
- Representativeness (The degree to which A resembles B)

- Systematically omit risk components

By allowing more than one evaluator to assess the risk associated with construction projects, a more objective and unbiased result can be obtained.
The fuzzy average operation for aggregate method that is known as the *Triangular Average Formula* is used to determine the mean of evaluator opinions. The triangular average formula is as follows:
Consider n evaluators and $A_i = (a_1^{(i)}, a_M^{(i)}, a_2^{(i)}), i = 1,...,n$. By using addition of triangular numbers and division by a real number, the triangular average (mean) A_{ave} will be as follow:

$$A_1 + A_2 = (a_1^{(1)} + a_M^{(1)} + a_2^{(1)}) + (a_1^{(2)} + a_M^{(2)} + a_2^{(2)})$$

$$= (a_1^{(1)} + a_1^{(2)}, a_M^{(1)} + a_M^{(2)}, a_2^{(1)} + a_2^{(2)})$$

$$\frac{A}{r} = \frac{1}{r}(a_1, a_M, a_2) = (\frac{a_1}{r}, \frac{a_M}{r}, \frac{a_2}{r})$$

$$A_{ave} = \frac{A_1 + ... + A_n}{n}$$

$$= \frac{(a_1^{(1)} + a_M^{(1)} + a_2^{(1)}) + ... + (a_1^{(n)} + a_M^{(n)} + a_2^{(n)})}{n}$$

$$= \frac{(\sum_{i=1}^{n} a_1^{(i)}, \sum_{i=1}^{n} a_M^{(i)}, \sum_{i=1}^{n} a_2^{(i)})}{n}$$

Which is a triangular number,

$$A_{ave} = (m_1, m_M, m_2) = (\frac{1}{n}\sum_{i=1}^{n} a_1^{(i)}, \frac{1}{n}\sum_{i=1}^{n} a_M^{(i)}, \frac{1}{n}\sum_{i=1}^{n} a_2^{(i)})$$

3.5. Fuzzy Weighted Average Computation; EFWA Algorithm

After computing the likelihood and severity of each risk from previous step, the EFWA algorithm will be applied. Based on (D.H.Lee, D.Park, 1997), the EFWA algorithm is as follow:
The basic idea of EFWA algorithm is to sort $a`s$ and $b`s$ in non-decreasing order and find two n-tuples $(d_1,...,d_i, c_{i+1},...,c_n)$ with the δ-threshold i and $(c_1,...,c_j, d_{i+1},...,d_n)$ with the ξ-threshold j ,which are the n-tuples such that
$f_L(d_1,...,d_i, c_{i+1},...,c_n) = \min f_L(w_1, w_2,..., w_n)$ and
$f_U(c_1,...,c_j, d_{i+1},...,d_n) = \max f_U(w_1, w_2,..., w_n)$.

EFWA Algorithm

1. Sort $a`s$ in non-decreasing order. Let $(a_1, a_2,..., a_n)$ be the resulting sequence. Let $first := 1$ and $last := n$.

2. Let δ-threshold:=$\lfloor (first + last)/2 \rfloor$. For each $i = 1,2,...,\delta$-threshold, let $e_i := d_i$ and for each $i = \delta$-threshold+1, ..., n, let $e_i := c_i$. For an n-tuples $S = (e_1, e_2, ..., e_n)$, evaluate $\delta_{S_{\delta-threshold}}$ and $\delta_{S_{(\delta-threshold+1)}}$.
3. If $\delta_{S_{\delta-threshold}}$ >0 and $\delta_{S_{(\delta-threshold+1)}} \le 0$ then $L = f_L(e_1, e_2, ..., e_n)$ and go to step 4; otherwise execute the following step.
   3.1. If $\delta_{S_{\delta-threshold}}$ >0, then $first := \delta$-threshold+1; otherwise $last := \delta$-threshold, and go to step 2.
4. Sort b`s in non-decreasing order. Let $(b_1, b_2, ..., b_n)$ be the resulting sequence. Let $first := 1$ and $last := n$.
5. Let ζ-threshold:=$\lfloor (first + last)/2 \rfloor$. For each $i = 1,2,...,\zeta$-threshold, let $e_i := c_i$ and for each $i = \zeta -$ threshold+1 ,..., n ,let $e_i := d_i$. For an n-tuples $S = (e_1, e_2, ..., e_n)$, evaluate $\zeta_{S_{\zeta-threshold}}$ and $\zeta_{S_{(\zeta-threshold+1)}}$.
6. If $\zeta_{S_{\zeta-threshold}} > 0$ and $\zeta_{S_{(\zeta-threshold+1)}} \le 0$ then $U = f_U(e_1, e_2, ..., e_n)$ and stop; otherwise execute the following step.
   6.1. If $\zeta_{S_{\zeta-threshold}} > 0$, then $first := \zeta$-threshold+1; otherwise $last := \zeta$-threshold, and go to step 5.

Where

$$\delta_{S_i} = \frac{(a_1 - a_i)e_1 + (a_2 - a_i)e_2 + ... + (a_n - a_i)e_n}{e_1 + e_2 + ... + e_n}$$

$$\zeta_{S_i} = \frac{(b_1 - b_i)e_1 + (b_2 - b_i)e_2 + ... + (b_n - b_i)e_n}{e_1 + e_2 + ... + e_n}$$

The fuzzy weighted average is:

$$\overline{W} = \frac{W_1 R_1 + ... + W_n R_n}{W_1 + ... + W_n}$$

3.6. Linguistic Approximation

The objective of this part is to find an appropriate natural language expression for the estimated fuzzy set. There are basically three techniques: Euclidean distance; Successive approximation; and Piecewise decomposition.

The Euclidean method is usually applied when the set of natural language expression is small (Roozbeh Kangari, Leland S.Riggs, 1989). It calculates the Euclidean distance from the given fuzzy set to each of the fuzzy sets representing the natural language expression. The distance between fuzzy sets X (unknown), and fuzzy set A (known) can be calculated as follows:

$$d(X, A) = \left\{ \sum_i [X_{(i)} - A_{(i)}]^2 \right\}^{1/2} \qquad i = 1 \text{ to } n$$

In which d =Euclidean Distance between two fuzzy sets; i =an integer between 1 and n; n = an integer that defines the highest value of the fuzzy set universe (K.W.Hipel, 1982).

The successive approximation method is applied when the set is large. This method assumes two close primary terms, then various expressions are applied to these two points in order to

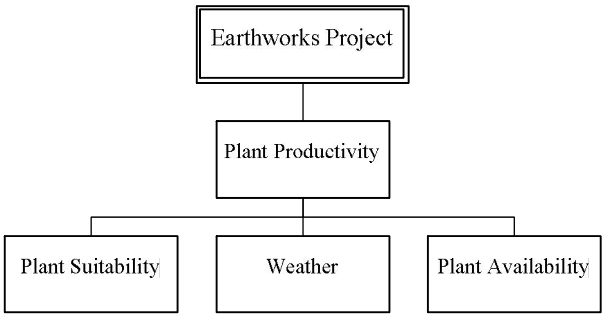
approximate the closest natural language expression. (D.P. Clements, 1971) (L.A. Neitzel and L.J.Hoffman, 1980).
The Piecewise decomposition method divides the linguistic variables into intervals. Then each interval is combined with one of the standard logical connectives to approximate the natural expression (Y.Leung, 1980).
In this research we use the Euclidean distance method because it is easy to understand and easy to implement on computers.
To map the resultant fuzzy interval back to one of the fuzzy numbers, a modified Euclidean approach which was proposed by Ross et. al. is used (E.W.T Ngai, F.K.T Wat, 2005). The difference measure d is given as follow:

$$d(X,A) = (\sum_{i}^{n}[\{X_{\min}(i) - A_{\min}(i)\}^2 + \{X_{\max}(i) - A_{\max}(i)\}^2])^{1/2}$$

Where i is the α value and n is the number of x-cut.
After obtaining the Euclidean distance, the model then assigns the appropriate natural language expression to the lowest Euclidean distance associated with fuzzy set X .

4. Fuzzy Risk Analysis Model Implementation

To implement mentioned fuzzy risk analysis model, assume an earthwork project as follow:



Plant suitability, availability and weather are assigned as risk factors (RF) for plant productivity. Assume three evaluators evaluate these risk factors as follow:

RF	Evaluator A		Evaluator B		Evaluator C	
	Likelihood	*Severity*	*Likelihood*	*Severity*	*Likelihood*	*Severity*
Suitability	Unlikely (0,0.25,0.5)	Moderate (0.25,0.5,0.75)	Medium (0.25,0.5,0.75)	Low (0,0.25,0.5)	Likely (0.5,0.75,1)	Minimal (0,0,0.25)
Weather	Medium (0.25,0.5,0.75)	Moderate (0.25,0.5,0.75)	Unlikely (0,0.25,0.5)	High (0.5,0.75,1)	Likely (0.5,0.75,1)	Critical (0.75,1,1)
Availability	Medium (0.25,0.5,0.75)	Low (0,0.25,0.5)	Likely (0.5,0.75,1)	Moderate (0.25,0.5,0.75)	Very Likely (0.75,1,1)	High (0.5,0.75,1)

Step1: Compute fuzzy average of risk factors

$$A_{ave} = (m_1, m_M, m_2) = (\frac{1}{n}\sum_{i=1}^{n} a_1^{(i)}, \frac{1}{n}\sum_{i=1}^{n} a_M^{(i)}, \frac{1}{n}\sum_{i=1}^{n} a_2^{(i)})$$

Fuzzy Average Results		
RF	Fuzzy Average of Likelihood (R)	Fuzzy Average of Severity (W)
Suitability	(0.25,0.5,0.75)	(0.125,0.375,0.5)
Weather	(0.125,0.5,0.75)	(0.5,0.75,0.916)
Availability	(0.5,0.75,0.916)	(0.25,0.5,0.75)

Step 2: Calculate fuzzy weighted average (FWA) by applying efficient fuzzy weighted average algorithm (EFWA)

For $\alpha = 0$ the intervals of R_i and W_i are:

$[a_1 = 0.25, b_1 = 0.75], [a_2 = 0.125, b_2 = 0.75], [a_3 = 0.5, b_3 = 0.916]$

$[c_1 = 0.125, d_1 = 0.5], [c_2 = 0.5, d_2 = 0.916], [c_3 = 0.25, d_3 = 0.75]$

EFWA Application:

Step 2-1-1:

$[a_1 = 0.125, b_1 = 0.75], [a_2 = 0.25, b_2 = 0.75], [a_3 = 0.5, b_3 = 0.916]$

$[c_1 = 0.5, d_1 = 0.916], [c_2 = 0.125, d_2 = 0.5], [c_3 = 0.25, d_3 = 0.75]$

$(a_1, a_2, a_3) = (0.125, 0.25, 0.5),\ first := 1, last := 3$

Step 2-1-2:

δ-threshold:= $\lfloor (1+3)/2 \rfloor = 2$

$S = (d_1, d_2, c_3) = (0.916, 0.5, 0.25) = (e_1, e_2, e_3)$

$$\delta_{S_2} = \frac{(0.125-0.25)\times 0.916 + (0.5-0.25)\times 0.25}{0.916+0.5+0.25} = -0.0312$$

$$\delta_{S_3} = \frac{(0.125-0.5)\times 0.916 + (0.25-0.5)\times 0.5}{0.916+0.5+0.25} = -0.2812$$

Step 2-1-3:

Since $\delta_{S_2} < 0$ and $\delta_{S_3} < 0$:

δ-threshold:= $\lfloor (1+2)/2 \rfloor = 1$

$S = (d_1, c_2, c_3) = (0.916, 0.125, 0.25) = (e_1, e_2, e_3)$

$$\delta_{S_1} = \frac{(0.25-0.125)\times 0.125 + (0.5-0.125)\times 0.25}{0.916+0.5+0.25} = 0.0656$$

$$\delta_{S_2} = \frac{(0.125-0.25)\times 0.916 + (0.5-0.125)\times 0.25}{0.916+0.5+0.25} = -0.0312$$

Since $\delta_{S_1} > 0$ and $\delta_{S_2} \leq 0$:

$L = f_L(0.5, 0.5, 0.75) = a_1 + \delta_{S_1} = 0.125 + 0.0656 = 0.1906$

$\text{Min} f_L = 0.1906$

Step 2-1-4:

$[a_1 = 0.125, b_1 = 0.75], [a_2 = 0.25, b_2 = 0.75], [a_3 = 0.5, b_3 = 0.916]$

$[c_1 = 0.5, d_1 = 0.916], [c_2 = 0.125, d_2 = 0.5], [c_3 = 0.25, d_3 = 0.75]$

$(b_1, b_2, b_3) = (0.75, 0.75, 0.916),\ first := 1, last := 3$

Step 2-1-5:

ζ -threshold=$\lfloor (1+3)/2 \rfloor = 2$

$S = (c_1, c_2, d_3) = (0.5, 0.125, 0.75) = (e_1, e_2, e_3)$

$$\zeta_{S_2} = \frac{(0.916 - 0.75) \times 0.75}{0.5 + 0.125 + 0.75} = 0.0905$$

$$\zeta_{S_3} = \frac{(0.75 - 0.916) \times 0.5 + (0.75 - 0.916) \times 0.125}{0.5 + 0.125 + 0.75} = -0.0754$$

Since $\zeta_{S_2} > 0$ and $\zeta_{S_3} \le 0$:

$U = f_U(0.916, 0.5, 0.25) = b_2 + \zeta_{S_2} = 0.75 + 0.0905 = 0.8405$

$\text{Max} f_U = 0.8405$

The interval for $\alpha = 0$ is (0.1906,0.8405) in which each point corresponds to the end point of the triangular that represents the membership function.

To obtain the center of the triangle, α will be assigned as 1.

For $\alpha = 1$ the intervals of R_i and W_i are:

$[a_1 = 0.5, b_1 = 0.5], [a_2 = 0.5, b_2 = 0.5], [a_3 = 0.75, b_3 = 0.75]$

$[c_1 = 0.375, d_1 = 0.375], [c_2 = 0.75, d_2 = 0.75], [c_3 = 0.5, d_3 = 0.5]$

Step 2-2-1:

$[a_1 = 0.5, b_1 = 0.5], [a_2 = 0.5, b_2 = 0.5], [a_3 = 0.75, b_3 = 0.75]$

$[c_1 = 0.375, d_1 = 0.375], [c_2 = 0.75, d_2 = 0.75], [c_3 = 0.5, d_3 = 0.5]$

$(a_1, a_2, a_3) = (0.5, 0.5, 0.75), \quad first := 1, last := 3$

Step 2-2-2:

δ -threshold:= $\lfloor (1+3)/2 \rfloor = 2$

$S = (d_1, d_2, c_3) = (0.375, 0.75, 0.5) = (e_1, e_2, e_3)$

$$\delta_{S_2} = \frac{(0.75 - 0.5) \times 0.5}{0.375 + 0.75 + 0.5} = 0.0769$$

$$\delta_{S_3} = \frac{(0.5 - 0.75) \times 0.375 + (0.5 - 0.75) \times 0.75}{0.375 + 0.75 + 0.5} = -0.1730$$

Since $\delta_{S_2} > 0$ and $\delta_{S_3} \le 0$:

$L = f_L(0.375, 0.75, 0.5) = a_2 + \delta_{S_2} = 0.5 + 0.0769 = 0.5769$

$\text{Min} f_L = 0.5769$

Step 2-2-3:

$[a_1 = 0.5, b_1 = 0.5], [a_2 = 0.5, b_2 = 0.5], [a_3 = 0.75, b_3 = 0.75]$

$[c_1 = 0.375, d_1 = 0.375], [c_2 = 0.75, d_2 = 0.75], [c_3 = 0.5, d_3 = 0.5]$

$(b_1, b_2, b_3) = (0.5, 0.5, 0.75), first := 1, last := 3$

Step 2-2-4:

ζ -threshold= $\lfloor (1+3)/2 \rfloor = 2$

$S = (c_1, c_2, d_3) = (0.375, 0.75, 0.5) = (e_1, e_2, e_3)$

$$\zeta_{S_2} = \frac{(0.75 - 0.5) \times 0.5}{0.375 + 0.75 + 0.5} = 0.0769$$

$$\zeta_{S_3} = \frac{(0.5 - 0.75) \times 0.375 + (0.5 - 0.75) \times 0.75}{0.375 + 0.75 + 0.5} = -0.1730$$

Since $\zeta_{S_2} > 0$ and $\zeta_{S_3} \leq 0$:

$U = f_U(0.375, 0.75, 0.5) = b_2 + \zeta_{S_2} = 0.5 + 0.0769 = 0.5769$

$\text{Max} f_U = 0.5769$

The interval for $\alpha = 1$ is (0.5769,0.5769) which corresponds to the center of the triangle.

Step 3: Euclidean distance computations

A Euclidean distance is used for mapping the resultant fuzzy intervals back to linguistic terms.

$$d(X, A) = (\sum_i^n [\{X_{\min}(i) - A_{\min}(i)\}^2 + \{X_{\max}(i) - A_{\max}(i)\}^2])^{1/2}$$

Following are the Euclidean distance measurements:

$d(X, Minimal) = 0.8472$

$d(X, Low) = 0.5091$

$d(X, Moderate) = 0.1328$

$d(X, High) = 0.3888$

$d(X, Critical) = 0.7193$

The closest Euclidean distance is 0.1328 which means that the risk of plant productivity is considered as *Moderate*.

5. Conclusion

Construction projects take place in a complex and challenging environment. High levels of risk are associated with this industry. A reliable way to analyze the associated risks is vital to make success. In this research we tried to propose a fuzzy risk analysis for construction projects. Although the computations involved in the model of the fuzzy risk analysis are tedious if performed manually, it is an easy task and the time for risk analysis can be significantly reduced. Construction project managers can predict the overall risk of the project before start the implementation. An overall risk index can be used as early indicators of project problems or potential difficulties. The proposed fuzzy risk analysis provides an effective, systematic and more natural way to analyze the associated risks. Evaluators can just simply use the risk evaluation checklist and use the linguistic terms to evaluate construction projects risk level. There are some limitations in this research. For example the membership functions were distributed by triangular fuzzy numbers. Various membership functions need to be estimated to be as realistic as possible.

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