

Optimum design of reinforced concrete waffle slabs

Alaa C. Galeb, Zainab F. Atiyah
Civil Engineering Department, University of Basrah, Iraq
alaagaleb@yahoo.com

ABSTRACT

This paper deals with the problem of optimum design of reinforced concrete (two-way ribbed) waffle slabs using genetic algorithms. Two case studies are discussed; the first is a waffle slab with solid heads, and the second is a waffle slab with band beams along column centerlines. Direct design method is used for the structural analysis and design of slabs. The cost function represents the cost of concrete, steel, and formwork for the slab. The design variables are taken as the effective depth of the slab, ribs width, the spacing between ribs, the top slab thickness, the area of flexural reinforcement at the moment critical sections, the band beams width, and the area of steel reinforcement of the beams. The constraints include the constraints on dimensions of the rib, and the constraints on the top slab thickness, the constraints on the areas of steel reinforcement to satisfy the flexural and the minimum area requirements, the constraints on the slab thickness to satisfy flexural behavior, accommodate reinforcement and provide adequate concrete cover, and the constraints on the longitudinal reinforcement of band beams. A computer program is written using MATLAB to perform the structural analysis and design of waffle slabs by the direct design method. The optimization process is carried out using the built-in genetic algorithm toolbox of MATLAB.

Keywords: Design, Optimisation, MATLAB, Genetic algorithm

1. Introduction

Waffle slab construction consists of rows of concrete joists at right angles to each other with solid heads at the column (needed for shear requirements) or with solid wide beam sections on the column centerlines for uniform depth construction. Fig. (1). Waffle slab construction allows a considerable reduction in dead load as compared to conventional flat slab construction since the slab thickness can be minimized due to the short span between the joists (PCA Notes on 318-05).

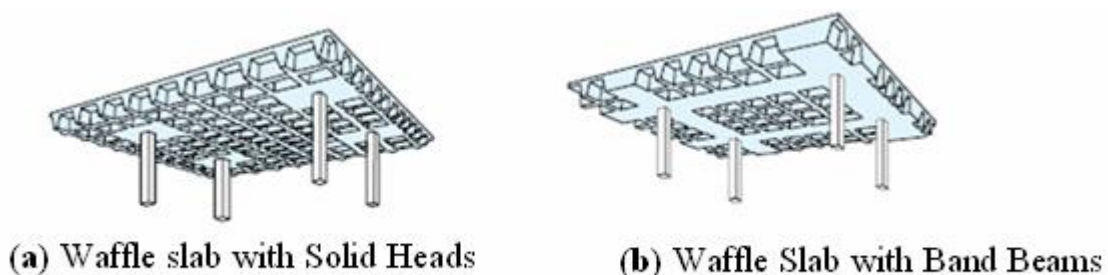


Figure 1: Waffle Slab Types

Cost optimum design of reinforced concrete structures is receiving more and more attention from the researchers. Ibrahim (1999) used mathematical programming techniques to minimize the cost of reinforced concrete T-beam floor. The floor system consisted of one way continuous slab and simply supported T-beam. A formulation based on an elastic analysis and the ultimate strength method of design with the consideration of serviceability constraints as per ACI 318-89 code is presented. Hadi (2001) presented the application of (GA) for the design of continuous reinforced concrete T and L beams based on the requirements of the Australian design standards for concrete structures, AS3600. Yokota et al. (2004) formulated an optimal T cross section design (OTCD) problem with singly reinforced concrete beams for a constrained ultimate strength in the concrete/steel and solved it directly by keeping the constraints based on an improved genetic algorithm (GA). They discussed the efficiency of the proposed method and the traditional method. Sahab et al. (2005) presented cost optimization of reinforced concrete flat slab buildings according to the British Code of Practice (BS8110). The objective function was the total cost of the building including the cost of floors, columns and foundations. Prasad et al. (2005) elaborated the results obtained from the analytical study carried out on waffle slab medium size floor system with a view to achieve the optimum dimensions of rib spacing, its depth and width. The waffle slab has been considered as monolithically connected to band beams. Feasibility of structural design of members has been ensured under the provision of IS: 456-2000. In this paper, the optimum design of reinforced concrete waffle slabs is sought using the simple genetic algorithm. A computer program is written using Matlab to formulate the problem and perform the structural analysis and design of those slabs by the direct design method. The cost function represents the cost of concrete, steel reinforcement and formwork. Specifying the optimum values of the various design variables is the main objective of this study. The problem is formulated based on the requirements of ACI 318-05 code and the ultimate strength design method. A number of examples were run to test the viability of the developed design formulation and all these examples proved that the method is versatile and leads to considerable savings in design.

1.1 Direct Design Method

The Direct Design Method (D.D.M) is an approximate procedure for analyzing two-way slab systems subjected to gravity loads only (PCA Notes on 318-05). Since it is approximate, the method is limited to slab systems meeting some limitations, these are:

1. There must be three or more continuous spans in each direction;
2. Slab panels must be rectangular with a ratio of longer to shorter span (centerline-to-centerline of supports) not greater than 2;
3. Successive span lengths (centerline-to-centerline of supports) in each direction must not differ by more than (1/3) of the longer span;
4. Columns must not be offset more than 10% of the span (in direction of offset) from either axis between centerlines of successive columns;
5. Loads must be uniformly distributed, with the unfactored or service live load not more than 2 times the unfactored or service dead load;
6. For two-way beam-supported slabs, relative stiffness of beams in two perpendicular directions must be

$$0.2 \leq \frac{\alpha_{f1} \ell_2^2}{\alpha_{f2} \ell_1^2} \leq 5.0$$

$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s}$$

Where:

α_{f1} = The ratio of flexural stiffness of beam to flexural stiffness of slab in direction ℓ_1

α_{f2} = The ratio of flexural stiffness of beam to flexural stiffness of slab in direction ℓ_2

ℓ_1 = Length of span in direction moments are being determined.

ℓ_2 = Length of span transverse to (ℓ_1)

E_{cb} = Modulus of elasticity of beam concrete

E_{cs} = Modulus of elasticity of slab concrete

I_b = Moment of inertia of uncracked beam

I_s = Moment of inertia of uncracked slab

7. Redistribution of negative moments is not permitted.

The Direct Design Method is essentially a three-step analysis procedure, involves (PCA Notes on 318-05):

(1) Determining the total factored static moment for each span,

$$M_o = \frac{q_u \ell_2 \ell_n^2}{8} \dots(1)$$

Where (q_u) is the factored combination of dead and live loads, ($q_u = 1.2w_d + 1.6w_l$).

(2) Dividing the total factored static moment between negative and positive moments within each span, as in Table (1).

(3) Distributing the negative and the positive moment to the column and the middle strips in the transverse direction (Tables 2 to 4).

Table 1: Distribution of Total Static Moment for an End Span

Factored Moment	(1)	(2)	(3)		(4)	(5)
	Slab Simply Supported on Concrete or Masonry Wall	Two-Way Beam-Supported Slabs	Flat Plates and Flat Slabs		Slab Monolithic with Concrete Wall	Slab Monolithic with Concrete Wall
			Without Edge Beam	With Edge Beam		
Interior Negative	0.75	0.70	0.70	0.70		0.65
Positive	0.63	0.57	0.52	0.50		0.35
Exterior Negative	0	0.16	0.26	0.30		0.65

analog to natural crossbreeding, or the copies may be modified, in an analog to genetic mutation. The new solutions are evaluated and added to the population, and low-quality solutions are deleted from the population to make room for new solutions. As this process of parent selection, copying, crossbreeding, and mutation is repeated, the members of the population tend to get better. When the algorithm is halted, the best member of the current population is taken as the solution to the problem posed. Then, the genetic algorithm loops over an iteration process to make the population evolve. Each iteration consists of the following steps:

- 1) **Selection:** the first step consists of selecting individuals for reproduction. This selection is done randomly with a probability depending on the relative fitness of the individuals so that best ones are often chosen for reproduction than poor ones.
- 2) **Reproduction:** in the second step, offspring are bred by the selected individuals. For generating new chromosomes, the algorithm can use both recombination and mutation.
- 3) **Evaluation:** then the fitness of the new chromosomes is evaluated.
- 4) **Replacement:** during the last step, individuals from the old population are killed and replaced by the new ones.

The algorithm is stopped when the population converges toward the optimal solution. The Genetic Algorithm process is described through the flowchart in Figure (3).

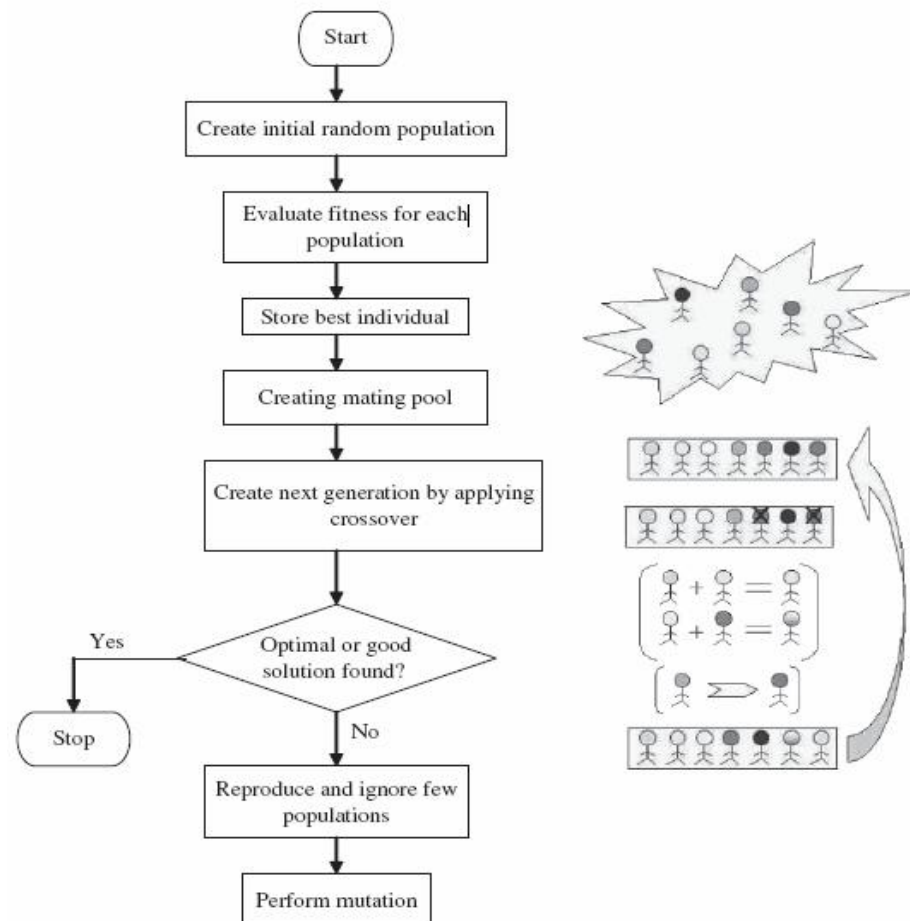


Figure 3: Genetic Algorithm Flowchart (Sivanandam, S.N., 2008)

2.1 Formulation of the Problem: Case (1) Waffle Slab with Solid Heads

The cost of materials (concrete and steel reinforcement) and formwork is considered as the objective function which should be minimized. The total cost of the slab can be stated as:

$$C = C_c \times (Q_c) + C_s \times (W_s) + C_f \times (A_f) \quad \dots \quad (2)$$

Where:

C = Total cost function

C_c = Cost of concrete per unit volume (I.D/m³)

C_s = Cost of steel per unit volume (I.D/ton)

C_f = Cost of formwork per unit area (I.D/m²)

Q_c = Concrete volume (m³)

W_s = Weight of steel (ton)

A_f = Surface area of the form (m²)

2.2 Formulation of the Constraints

The following constraints are considered in this paper:

1- Ribs shall not be less than 100 mm in width, (American Concrete Institute, 2005) i.e.,

$$b_r \geq 100mm$$

$$g_1 = 0.1 - b_r \leq 0$$

2- Ribs shall have a depth not more than 3.5 times the minimum width of rib, (American Concrete Institute, 2005) i.e.,

$$((1 + t_t) \times d - h_s) \leq 3.5 \times b_r$$

$$g_2 = ((1 + t_t) \times d - h_s) - 3.5 \times b_r \leq 0$$

3- Clear spacing between ribs shall not exceed 750 mm (American Concrete Institute, 2005), this gives:

$$(S - b_r) \leq 750mm$$

$$g_3 = (S - b_r) - 0.75 \leq 0$$

where

t_t = ratio of concrete cover to the effective depth of the slab.

4- When permanent burned clay or concrete tile fillers of material having a compressive strength at least equal to f_c' in the joists are used, the top slab thickness shall be not less than one-twelfth the clear distance between ribs, nor less than 40 mm (American Concrete Institute, 2005), i.e.,

$$h_s \geq \frac{(S - b_r)}{12}$$

$$g_4 = \frac{(S - b_r)}{12} - h_s \leq 0$$

and,

$$h_s \geq 40mm$$

$$g_5 = 1 - \frac{h_s}{0.04} \leq 0$$

6- For slabs without interior beam spanning between the supports, the minimum thickness shall be in accordance with the provisions of Table (5) and shall not be less than (100 mm) (American Concrete Institute, 2005), i.e.,

$$h \geq 100mm$$

$$0.100 \leq (1 + t_i) \times d$$

$$g_6 = 1 - \frac{(1 + t_i) \times d}{0.100} \leq 0$$

The minimum slab thickness for an exterior panel with drop panel, can be found from Table (5) using linear interpolation as $(\ell_n / 32)$, so,

$$g_7 = \frac{\ell_n}{32} - (1 + t_i) \times d \leq 0$$

Table 5: Minimum Thickness of Slabs without Interior Beams (American Concrete Institute, 2005)

f_y (MPa)	Without drop panels			With drop panels		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beam		Without edge beams	With edge beams	
280	$\frac{\ell_n}{33}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{40}$	$\frac{\ell_n}{40}$
420	$\frac{\ell_n}{30}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$
520	$\frac{\ell_n}{28}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{34}$	$\frac{\ell_n}{34}$

7- At every section of a flexural member where tensile reinforcement is required, the area of steel reinforcement shall not be less than $(A_{S\min})$ given by:

$$A_{S\min} = \frac{0.25 \times \sqrt{f_c'}}{f_y} \times b_r \times d$$

or

$$A_{S\min} = \frac{1.4 \times b_r \times d}{f_y}$$

$$A_S \geq A_{S\min}$$

$$A_{S\min} - A_S \leq 0$$

$$g_8 = 1 - \frac{A_s \times f_y}{0.25 \times \sqrt{f'_c} \times b_r \times d} \leq 0$$

8- Sections are tension-controlled if the net tensile strain in the extreme tensile steel (ε_t) is equal to or greater than 0.005 when the concrete in compression reaches its assumed strain limit of 0.003 (American Concrete Institute, 2005).

$$\varepsilon_t > 0.005$$

$$\varepsilon_t = \left(\frac{0.003}{c_t} \right) \times d - 0.003$$

$$c_t = \left(\frac{A_{Si} \times f_y}{0.85 \times f'_c \times b} \right) \beta_1$$

$$\left(\frac{0.003}{\left(\frac{A_{Si} \times f_y}{0.85 \times \beta_1 \times f'_c \times b} \right)} \right) \times d - 0.003 - 0.005 > 0$$

$$g_9 = \left(\frac{0.003}{\left(\frac{A_{Si} \times f_y}{0.85 \times \beta_1 \times f'_c \times b} \right)} \right) \times d - 0.008 > 0$$

$$g_9 = 0.008 - \left(\frac{0.003}{\left(\frac{A_{Si} \times f_y}{0.85 \times \beta_1 \times f'_c \times b} \right)} \right) \times d \leq 0$$

9- The moment capacity of the section must be greater than the applied moment i.e.,

$$M \geq \frac{m_i}{S}$$

$$\frac{m_i}{S} \leq 0.85 \times f'_c \times a \times b_r \times \left(d - \frac{a}{2} \right) + 0.85 \times f'_c \times h_s \times (b - b_r) \times \left(d - \frac{h_s}{2} \right)$$

$$a = \frac{A_{Si} \times f_y}{0.85 \times f'_c \times b_r}$$

$$\frac{m_i}{S} \leq \left[A_{Si} \times f_y \times \left(d - \frac{A_{Si} \times f_y}{0.85 \times f'_c \times b_r \times 2} \right) + 0.85 \times f'_c \times h_s \times (b - b_r) \times \left(d - \frac{h_s}{2} \right) \right]$$

$$g_{10} = 1 - [A_{si} \times f_y \times \left(d - \frac{A_{si} \times f_y}{1.7 \times f'_c \times b_r} \right) + 0.85 \times f'_c \times h_s \times (b - b_r) \times \left(d - \frac{h_s}{2} \right)] \times \frac{S}{m_i} \leq 0$$

where,

$$m_i = m_{c1}, m_{c2}, m_{m1}, m_{m2}$$

m_{c1} = Negative moment in column strip

m_{c2} = Positive moment in column strip

m_{m1} = Negative moment in middle strip

m_{m2} = Positive moment in middle strip

$$A_{si} = A_{src}^+, A_{src}^-, A_{srm}^+, A_{srm}^-$$

A_{src}^- = negative reinforcement in the column strip.

A_{src}^+ = positive reinforcement in the column strip.

A_{srm}^- = negative reinforcement in the middle strip.

A_{srm}^+ = positive reinforcement in the middle strip.

b = Flange width

10- It is assumed in this study that the top slab thickness has a maximum value of not more 100 mm (which is found in many literature), i.e.,

$$h_s \leq 100mm$$

$$g_{11} = 1 - \frac{0.1}{h_s} \leq 0$$

11- Punching Shear Constraint:

The two-way shear strength of slab section must be greater than the applied shear stress at the critical section (at distance $d/2$ from the face of the support), this gives:

$$\frac{\phi}{3} \times \sqrt{f'_c} \times b_o \times d \geq V_u$$

$$\frac{0.7}{3} \times \sqrt{f'_c} \times [(c + d) \times 4 \times d] \geq [(\ell_1 \times \ell_2 - (c + d)^2) \times \{1.2 \times w_d + 1.6 \times w_\ell\}]$$

$$w_d = d_l + [(S \times h_s + b_r \times ((1 + t_t) \times d - h_s)) \times \ell_1 \times 24 \times 2 \times \left(\frac{\ell_1}{S} + 1 \right)] / (\ell_1 \times \ell_2)$$

$$g_{12} = 1 - \frac{\frac{0.7}{3} \times \sqrt{f'_c} \times [(c + d) \times 4 \times d]}{(\ell_1 \times \ell_2 - (c + d)^2) \times \left\{ \begin{array}{l} 1.2 \times (d_l + [(S \times h_s + b_r \times ((1 + t_t) \times d - h_s)) \times \ell_1 \times 24 \times 2 \times \left(\frac{\ell_1}{S} + 1 \right)]) \times \\ \ell_1 \times 24 \times 2 \times \left(\frac{\ell_1}{S} + 1 \right)] / (\ell_1 \times \ell_2) + 1.6 \times w_\ell \end{array} \right\}} \leq 0$$

where,

c = Dimension of column (m)

Now, the optimization problem can be stated as follows: Find the values of the design variables $(d, b_r, S, A_{src}^-, A_{src}^+, A_{srm}^-, A_{srm}^+, h_s)$ which minimize the cost function (C) under the constraints $(g_1$ to $g_{12})$ stated above. To solve this constrained optimization problem, the Matlab Toolbox of genetic algorithm is used. The constraints are taken into consideration by converting the above constrained nonlinear problem to an unconstrained one using penalty function. This is done automatically by the toolbox which provides a penalty parameter with default value of (100). In this study, this default penalty parameter value is used.

Case (2) Waffle Slab with Band Beams along Column Centerlines

As in the previous case, the total cost function is stated as:

$$C = C_c \times (Q_c) + C_s \times (W_s) + C_f \times (A_f) \quad \dots \quad (3)$$

The constraints are the same as those for the previous case $(g_1$ to $g_{11})$ (no punching shear constraint is considered here) besides that related to the band beam and can be derived as follows:

$$2 \times \ell_b \geq c$$

$$c \leq 2 \times \ell_b$$

$$g_{12} = 1 - \frac{2 \times \ell_b}{c} \leq 0$$

$$g_{13} = 1 - \left(\frac{A_{sb} \times f_y \times (d - (A_{sb} \times f_y / 1.7 \times f'_c \times b_r))}{0.85 \times f'_c \times h_s \times (b - b_r) \times (d - h_s / 2)} \right) \times \frac{S}{m_i} \leq 0$$

Where,

ℓ_b = Half width of the band beam.

W_{sb} = Weight of steel reinforcement of the beam (ton)

$$A_{sb} = A_{sb}^-, A_{sb}^+$$

A_{sb}^- = Negative area of longitudinal steel in band beam (m^2)

A_{sb}^+ = Positive area of longitudinal steel in band beam (m^2)

$$g_{14} = 1 - \frac{A_{sb}^- \times f_y}{0.25 \times \sqrt{f'_c} \times 2 \times \ell_b \times d} \leq 0$$

$$g_{15} = 1 - \frac{0.31875 \times \beta_1 \times f'_c \times b \times d}{A_{sb}^- \times f_y} \leq 0$$

Examples: (1) Waffle Slab with Solid Heads

In this application, a waffle slab with solid heads, consists of three by three square panels, is considered. The span length (ℓ) is 7m, the solid heads are square of 1.2m×1.2m and the

columns are of square cross-sections of 0.6m. The slab is subjected to a total load representing its self-weight and a dead load of 3kN/m^2 , and a live load of 4kN/m^2 . Other data are: the cylinder concrete compressive strength (f'_c) =30 MPa, the yield stress of steel (f_y) =460 MPa, the cost of concrete per unit volume (C_c) =175000 (I.D/ m^3), the cost of steel per unit volume (C_s) =1250000 (I.D/ton) and the cost of formwork (C_f) =10000 (I.D/ m^2). Table (6) shows the initial population and final results of this application.

Table 6: Initial Population and Final Results for a Waffle Slab with Solid Heads

Values	d (m)	b_r (m)	S (m)	A_{src}^- (m^2)	A_{srm}^- (m^2)	A_{src}^+ (m^2)	A_{srm}^+ (m^2)	h_s (m)	C (I.D)
Initial Population	0.850	0.716	1.460	0.007	0.004000	0.004000	0.004	0.070	167080967
	0.557	0.196	0.938	0.002	0.002000	0.004000	0.002	0.097	124619230
	0.591	0.540	1.256	0.002	0.004000	0.002000	0.002	0.081	123382514
	0.525	0.349	1.086	0.003	0.001000	0.001000	0.001	0.080	104342236
	0.503	0.793	1.483	0.002	0.002000	0.002000	0.002	0.059	108089190
	0.474	0.587	1.297	0.006	0.004000	0.004000	0.001	0.067	130551443
	0.591	0.733	1.482	0.006	0.002000	0.004000	0.002	0.093	131527013
	0.791	0.782	1.528	0.009	0.003000	0.005000	0.004	0.091	166576758
	0.750	0.724	1.472	0.007	0.002000	0.004000	0.002	0.070	147007115
	0.450	0.350	1.100	0.005	0.003000	0.004000	0.001	0.065	125993144
	0.400	0.300	0.900	0.004	0.002000	0.006000	0.003	0.099	145886909
	0.450	0.250	0.850	0.005	0.003000	0.002000	0.005	0.095	152735016
	0.500	0.350	1.000	0.004	0.005000	0.006000	0.003	0.097	158704115
	0.380	0.350	1.100	0.002	0.003000	0.002000	0.001	0.100	102240662
	0.597	0.541	1.290	0.002	0.004000	0.005000	0.002	0.075	132255110
	0.650	0.596	1.343	0.007	0.004000	0.006000	0.002	0.070	154646904
	0.650	0.548	1.289	0.002	0.004000	0.002000	0.002	0.070	126399628
	0.700	0.550	1.200	0.003	0.003000	0.006000	0.002	0.055	149332855
0.700	0.551	1.200	0.006	0.002000	0.002000	0.002	0.055	142163766	
0.600	0.641	1.299	0.004	0.003000	0.002000	0.002	0.055	128497804	
Final	0.310	0.392	1.140	0.002	0.000457	0.000499	0.001	0.091	81365709

-Population size=20
-No. of generations=93

3. Discussions

3.1 Population Size

The default initial population size in the Genetic Algorithm Matlab Toolbox is 20 individuals. In order to explain the effect of population size on the optimum solution, various values of population size are used. Figure (4) shows the effect of population size on the minimum cost of a waffle slab. It can be noted that the increasing in population size gives smaller values of minimum cost. Table (7) presents the optimum values of the design variables for various values of population size. This table shows that the increasing in population size (up to 80 individuals) leads the solution towards the optimum solution in a less number of generations

(iterations). Any further increasing in the population size will increase the required number of generations.

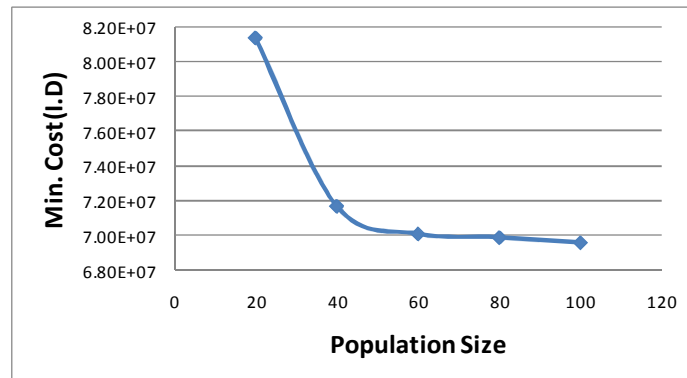


Figure 4: Effect of Population Size on the Minimum Cost in Waffle Slab with Solid Heads

Table 7: Optimum Values of the Design Variables for Various Values of Population Size for a Waffle Slab with Solid Heads

Pop. Size	C (I.D) E+07	No. of Gen.	d (m)	b_r (m)	S (m)	A_{src}^- (m ²) E-04	A_{srm}^- (m ²) E-04	A_{src}^+ (m ²) E-04	A_{srm}^+ (m ²) E-04	h_s (m)
20	8.14	93	0.310	0.392	1.140	20.0	4.57	4.99	10.0	0.091
40	7.17	54	0.317	0.286	1.033	4.75	9.70	0.147	0.721	0.075
60	7.01	22	0.226	0.361	1.107	10.0	4.28	2.72	8.48	0.083
80	6.99	13	0.243	0.415	1.164	10.0	1.18	1.45	10.0	0.072
100	6.96	14	0.234	0.469	1.219	10.0	5.03	7.43	1.46	0.067

-Span Length =7m

2) Span Length

Table (8) shows the optimum values of the design variables and the total cost of the slab for various values of span length.

Table 8: Optimum Design Variables Values for Various Span Lengths of a Waffle Slab with Solid Heads

Span Length (m)	C (I.D) E+07	d (m)	b_r (m)	S (m)	A_{src}^- (m ²) E-04	A_{srm}^- (m ²) E-04	A_{src}^+ (m ²) E-04	A_{srm}^+ (m ²) E-04	h_s (m)
7	7.17	0.317	0.286	1.033	4.75	9.70	0.147	0.721	0.075
8	9.47	0.286	0.350	1.100	1.34	3.42	6.33	10.0	0.065
10	18.5	0.450	0.337	1.086	20.00	1.00	5.78	4.70	0.065
12	29.3	0.515	0.191	0.939	20.00	2.00	4.31	5.14	0.068
15	48.5	0.810	0.236	0.986	6.40	1.00	7.35	2.80	0.066

- Population Size =40

3) Effective Depth

Table (9) shows the optimum values of design variables and the (effective depth/span length) ratio. It can be noted that for the optimum design, the ratio of effective depth to span length (d/ℓ) should be between (1/28-1/19). Also it can be noted from this table that the effective depth values for solution are ranged from 286 mm up to 810 mm in comparison with the practical range found in the literatures for overall depth which is (300-600) mm for span length up to 15 m (Cement and Concrete Association of Australia, 2003).

Table 9: The Optimum Ratio of (Effective Depth / Span Length) for a Waffle Slab with Solid Heads

Span Length (m)	C (I.D) E+07	d (m)	b_r (m)	S (m)	h_s (m)	d/ℓ
7	7.17	0.317	0.286	1.033	0.075	1/22
8	9.47	0.286	0.350	1.100	0.065	1/28
10	18.4	0.450	0.337	1.086	0.065	1/22
12	29.3	0.515	0.191	0.939	0.068	1/23
15	48.5	0.810	0.236	0.986	0.066	1/19

4) Clear Spacing

Table (10) presents the optimum values of the center-to-center spacing between ribs (S) for various values of span length. Constraint g_3 states that the clear spacing between ribs ($S - b_r$) should be less than 750 mm. It may be noted that this constraint is always active (i.e. the clear spacing ≤ 750 mm controls the solution). It may also be noted that the rib spacing corresponding to the optimum values of the design variables are to be within the range (600-1500) mm which defined as practical limit in literatures.

Table 10: Optimum Values of Design Variables and Clear Spacing between Ribs for Various Values of Span Length for a Waffle Slab with Solid Heads

Span (m)	C (I.D) E+07	d (m)	b_r (m)	S (m)	A_{src}^- (m ²) E-04	A_{srm}^- (m ²) E-04	A_{src}^+ (m ²) E-04	A_{srm}^+ (m ²) E-04	h_s (m)	$S - b_r$ (m)	S/ℓ
7	7.17	0.317	0.286	1.033	4.75	9.70	0.147	0.721	0.075	0.75	0.147
8	9.47	0.286	0.350	1.100	1.34	3.42	6.33	10.0	0.065	0.75	0.137
10	18.4	0.450	0.337	1.086	20.0	10.0	5.78	4.70	0.065	0.75	0.108
12	29.3	0.515	0.191	0.939	20.0	20.0	4.31	5.14	0.068	0.75	0.0783
15	48.5	0.810	0.236	0.986	6.40	10.0	7.35	2.80	0.066	0.75	0.0657

5) Rib Width

It may be noted, from Table (7), that the rib width values corresponding to the optimum cost are ranged from 191 mm to 350 mm, while the practical values found in the literatures are within the range (125-200) mm (Cement and Concrete Association of Australia, 2003).

6) Top Slab Depth

As stated the literature, the top slab depth typically varies from 75 to 125 mm. From table (8) it may be noted that the optimum values of top slab depth lie within the range (65-75) mm.

7) The Effect of Unit Costs

In order to illustrate the effect of the unit costs of the concrete and steel, the cost function can be written in the following form:

$$\frac{C}{C_s} = \frac{C_c}{C_s} \times (Q_c) + (W_s) + \frac{C_f}{C_s} \times (A_f) \quad \dots \quad (4)$$

Table (11) shows that the increase of the ratio (C_c/C_s) leads to decrease the optimum values of the rib cross-sectional area (A_c) and decrease the rib spacing (S) and increase the area of steel reinforcement. This will cause the volume of concrete (the material of higher cost) to decrease.

Table 11: Effect of Increasing the Ratio (C_c/C_s) on the Optimum Design for a Waffle Slab with Solid Heads

C_c/C_s	C (I.D) E+07	d (m)	b_r (m)	S (m)	A_{src}^- (m ²) E-04	A_{srm}^- (m ²) E-04	A_{src}^+ (m ²) E-04	A_{srm}^+ (m ²) E-04	h_s (m)	No. of Ribs
0.14	7.17	0.317	0.286	1.033	4.75	9.70	0.147	0.721	0.075	8
0.16	7.43	0.261	0.168	0.899	3.74	6.30	10.0	3.78	0.097	9
0.18	7.44	0.342	0.104	0.854	3.57	2.23	2.72	4.54	0.097	10
0.20	8.32	0.296	0.107	0.854	9.07	0.516	20.0	10.0	0.067	10
0.22	8.93	0.269	0.108	0.857	20.0	10.0	526	10.0	0.085	10

-Population Size =40

-Span Length =7m

8) Cost of Formwork

In order to study the effect of cost of formwork on the optimum solution, the cost function can be written in the following form:

$$C = C_c \times (Q_c) + C_s \times (W_s)$$

Table 12 shows the optimum values of design variables and the total cost of the slab without cost of formwork.

Table 13 summarizes the results obtained when the cost of formwork is included or not. It can be noted that the value cost of formwork is (0.85-1.37) of the total slab cost.

Table 12: Optimum Solution without the Cost of Formwork for a Waffle Slab with Solid Heads

Span Length (m)	Cost without Formwork (I.D) E+07	d (m)	b_r (m)	S (m)	A_{src}^- (m ²) E-04	A_{srm}^- (m ²) E-04	A_{src}^+ (m ²) E-04	A_{srm}^+ (m ²) E-04	h_s (m)
7	3.69	0.229	0.269	1.014	20.0	10.0	10.0	10.0	0.064
8	4.96E	0.313	0.487	1.235	10.0	10.0	10.0	7.98	0.069
10	9.18	0.525	0.285	1.034	5.29	10.0	10.0	10.0	0.080
12	15.8	0.631	0.300	1.047	20.0	5.51	3.01	10.0	0.083
15	20.5	0.809	0.235	0.985	2.94	1.28	2.02	5.82	0.066

Table 13: Effect of Formwork Cost for a Waffle Slab with Solid Heads

Span Length (m)	Including Formwork Cost					Not Including Formwork Cost					Cost with Formwork Cost/ Cost without Formwork Cost
	d (m)	b_r (m)	S (m)	h_s (m)	Cost (I.D) E+07	d (m)	b_r (m)	S (m)	h_s (m)	Cost (I.D) E+07	
7	0.317	0.286	1.033	0.075	7.17	0.229	0.269	1.014	0.064	3.69	1.94
8	0.286	0.350	1.100	0.065	9.47	0.313	0.487	1.235	0.069	4.96	1.91
10	0.450	0.337	1.086	0.065	18.5	0.525	0.285	1.034	0.080	9.18	2.02
12	0.515	0.191	0.939	0.068	29.3	0.631	0.300	1.047	0.083	15.8	1.85
15	0.810	0.236	0.986	0.066	48.5	0.809	0.235	0.985	0.066	20.5	2.37

Example (2): Waffle Slab with Band Beams along Column Centerlines

Waffle slab with band beams along column centerlines consist of three by three square panels of span (ℓ) length 7m and the columns are of square cross-section of 0.6m. The slab is subjected to a total load representing its self-weight and a dead load of 3kN/m^2 , and a live load of 4kN/m^2 . Other data are: the cylinder concrete compressive strength (f'_c)=30 MPa, the yield stress of steel (f_y)=460 MPa, the cost of concrete per unit volume (C_c)=175000 (I.D/m³), and the cost of steel per unit volume (C_s)=1250000 (I.D/ton) and the cost of formwork (C_f)=10000 (I.D/m²).

Table (14) shows the initial population and final results for waffle slab with band beams along column centerlines.

Table 14: Initial Population and Final Results for a Waffle Slab with Band Beams

Values	d (m)	b_r (m)	S (m)	A_{src}^- (m ²) E-03	A_{srm}^- (m ²) E-03	A_{src}^+ (m ²) E-03	A_{srm}^+ (m ²) E-03	h_s (m)	$2\ell_b$ (m)	A_{sp}^- (m ²) E-03	A_{sp}^+ (m ²) E-03	C (I.D) E+13
Initial Population	0.850	0.716	1.460	7	4	4	4	0.070	0.40	6	2	3.242
	0.557	0.196	0.938	2	2	4	2	0.097	0.60	4	1	2.866
	0.591	0.540	1.256	2	4	2	2	0.081	0.50	6	8	2.890
	0.525	0.349	1.086	3	1	1	1	0.080	0.30	5	1	2.120
	0.503	0.793	1.483	2	2	2	2	0.059	0.20	3	0.9	2.051
	0.474	0.587	1.297	6	4	4	1	0.067	0.16	2	2	2.455
	0.591	0.733	1.482	6	2	4	2	0.093	0.80	1	6	3.211
	0.791	0.782	1.528	9	3	5	4	0.091	0.16	8	0.4	3.220
	0.750	0.724	1.472	7	2	4	2	0.070	0.90	5	0.6	3.521
	0.500	0.350	1.000	4	5	6	3	0.097	0.23	8	1	3.290
	0.380	0.350	1.100	2	3	2	1	0.100	0.50	1	0.8	2.255
	0.597	0.541	1.290	2	4	5	2	0.075	0.70	0.9	1	2.962
	0.650	0.596	1.343	7	4	6	2	0.070	0.30	2	8	3.289
	0.650	0.548	1.289	2	4	2	2	0.070	0.90	6	1	3.280
	0.700	0.551	1.200	6	2	2	2	0.055	0.26	9.7	2	3.098
0.600	0.641	1.299	4	3	2	2	0.055	0.90	1	6	3.268	
Final	0.557	0.495	1.245	2	3	2	2	0.072	0.60	0.4	1	1.814

-No. of generations=61

4. Discussions

1) Effective Depth

Table (15) shows the optimum values of design variables and the (effective depth/span length) ratio. It can be noted that for the optimum design, the ratio of effective depth to span length (d/ℓ) should be between (1/33-1/18). Also it can be noted from this table that the effective depth values for solution are ranged from 280 mm up to 450 mm in comparison with the practical range found in the literatures for overall depth which is (300-600) mm for span length up to 15 m (Cement and Concrete Association of Australia, 2003)

Table 15: The Optimum Ratio of (Effective Depth / Span Length) for a Waffle Slab with Band Beams

Span Length (m)	C (I.D)	d (m)	b_r (m)	S (m)	d/ℓ
7	1.52E+08	0.380	0.388	1.136	1/18
8	1.65E+08	0.280	0.450	1.200	1/29
10	2.48E+08	0.380	0.478	1.226	1/26
12	3.24E+08	0.380	0.486	1.235	1/32
15	5.34E+08	0.450	0.484	1.233	1/33

2) Clear Spacing

Table (16) presents the optimum values of center-to-center spacing between ribs (S) for various values of span length. Constraint g_3 states that the clear spacing between ribs ($S - b_r$) should be less than 750mm. It may be noted that this constraint is always active (i.e. the clear spacing ≤ 750 mm controls the solution). It may also be noted that the rib spacing corresponding to the optimum values of the design variables are to be within the range (600-1500) mm which defined as practical limit in literatures.

Table 16: The Optimum Values of Design Variables and Clear Spacing between Ribs for Various Values of Span Length for a Waffle Slab with Band Beams

Span Length (m)	C (I.D) E+08	d (m)	b_r (m)	S (m)	$S - b_r$ (m)	S/ℓ
7	1.52	0.380	0.388	1.136	0.75	0.1623
8	1.65	0.280	0.450	1.200	0.75	0.1500
10	2.48	0.380	0.478	1.226	0.75	0.1226
12	3.24	0.380	0.486	1.235	0.75	0.1029
15	5.34	0.450	0.484	1.233	0.75	0.0822

3) Rib Width

It may be noted, from Table (16), that the rib width values corresponding to the optimum cost are ranged from 338 mm to 486 mm, while the practical values found in the literatures are within the range (125-200) mm (Cement and Concrete Association of Australia, 2003).

4) Top Slab Depth

The top slab depth typically (found in the literature) varies from 75 to 125 mm. From table (14) it may be noted that the optimum values of top slab depth lie within the range (62-72) mm.

Table 17: Optimum Values of the Design Variables for Various Values of Span Length for a Waffle Slab with Band Beams

Span Length (m)	C (I.D)	d (m)	b_r (m)	S (m)	h_s (m)	$2\ell_b$ (m)	A_{sp}^- (m ²)	A_{sp}^+ (m ²)
7	1.52E+08	0.380	0.388	1.136	0.065	0.60	5.00E-04	1.00E-03
8	1.65E+08	0.280	0.450	1.200	0.064	0.60	6.00E-04	4.00E-03
10	2.48E+08	0.380	0.478	1.226	0.072	0.60	9.31E-05	1.00E-03
12	3.24E+08	0.380	0.486	1.235	0.065	0.60	9.00E-04	1.00E-03
15	5.34E+08	0.450	0.484	1.233	0.062	0.60	9.39E-04	3.00E-03

4.1 Comparison Study

Table (18) summarizes the results obtained from the two case studies that discussed for various values of span length. It may be noted that the total cost of waffle slab with band beams is higher than that with solid head for slabs with the same span length. The ratio of the

total costs is found to be within the range (1.10-2.12). It may also be noted that the cost ratio decreases as the span length increases.

Table 18: Optimum Comparison between Results of the Two Cases Study

Span Length (m)	Waffle Slab with Solid Heads				Waffle Slab with Band Beams				C_2/C_1
	d (m)	b_r (m)	S (m)	C_1 (I.D)	d (m)	b_r (m)	S (m)	C_2 (I.D)	
7	0.317	0.286	1.033	7.17E+07	0.380	0.388	1.136	1.52E+08	2.12
8	0.286	0.350	1.100	9.47E+07	0.280	0.450	1.200	1.65E+08	1.74
10	0.450	0.337	1.086	1.85E+08	0.380	0.478	1.226	2.48E+08	1.34
12	0.515	0.191	0.939	2.93E+08	0.380	0.486	1.235	3.24E+08	1.11
15	0.810	0.236	0.986	4.85E+08	0.450	0.484	1.233	5.34E+08	1.10

5. Conclusions

The following conclusions may be drawn from the present study:

1. The population size affects the obtained optimum solution. The increasing in population size enhances the optimum value of the total cost. This is because the diversity of large size population.
2. The increasing in population size (sometimes up to a certain limit) gives the final optimum solution in a less number of generations.
3. For waffle slab with solid heads, the ratio of effective depth to span length (d/ℓ) should be (1/28-1/19) to get the optimum design, while for waffle slab with band beams along columns centerlines, it should be (1/33-1/18).
4. The clear spacing corresponding to the optimum values of the design variables is found to be within the range (600-1500) mm which defined as practical limit in literatures.
5. The center-to-center spacing between ribs is found to be (6.57%-14.76%) of the span length to get the optimum total cost of waffle slab with solid heads, while it should be (8.22%-16.23%) of the span length for optimum design of waffle slab with band beams.
6. The rib width values, corresponding to the optimum cost of waffle slab with span length less than 15 m, are ranged from 191 mm to 350 mm for slabs with solid heads and from 388 mm to 486 mm for slabs with band beams. The practical values found in the literatures are within the range (125-200) mm for span length less than 15 m. This means that the optimum solution gives higher rib width values than the practical limit, so, it can be concluded that the optimum solution tends to get a flat plate.
7. The optimum values of the top slab depth are found to be within the range (65-75) mm for slab with solid heads and between (62-72) mm for slab with band beams, while the practical limit found in the literatures are ranged from 75 mm to 125 mm.
8. The increasing in the ratio of concrete cost relative to the steel cost causes a decreasing in the rib spacing and the cross-sectional area of the ribs. While the increasing in the steel unit cost relative to the concrete unit cost causes an increasing in the cross-sectional area of the ribs.
9. The cost of formwork of the slab is found to be (85%-137%) of the total slab cost for slabs with solid heads and for slabs with band beams is (30%-64%).

-
10. For same span length, it is found that the total cost of waffle slab with band beams along columns centerlines is (10%-112%) higher than the total cost of waffle slab with solid heads.

6. References

1. American Concrete Institute, "Building Code Requirements for Structural Concrete & PCA Notes on 318-05" American Concrete Institute, Farmington Hills, Michigan, 2005.
2. Ibrahim, N.A., "Optimal Design of Reinforced Concrete T-Beam Floors". M.Sc. Thesis, University of Basrah, Iraq, 1999.
3. Hadi, M.N.S., "Optimum Design of Reinforced Concrete Continuous Beams by Genetic Algorithms". Proceedings of the Eighth International Conference on Civil and Structural Engineering Computing, 2001.
4. Yokota, T., Wada, S., Taguchi, T., and Gen, M., "GA-Based Method for a Single Reinforce Concrete Beam Optimal T Cross-Section Design Problem using the Ultimate Strength". Proceedings of the Fifth Asia Pacific Industrial Engineering and Management Systems Conference, 2004.
5. Sahab, M.G., Ashour, A.F., and Toropov., V.V., "Cost optimization of reinforced concrete flat slab buildings". Engineering Structures, Vol.27, pp.313-322, 2005.
6. Prasad, J., Chander, S., Ahuja, A.K., "Optimum Dimensions of Waffle Slab for Medium Size Floors". Asian Journal of Civil Engineering (Building and Housing), Vol.6, NO.3, pp 183-197, (2005).
7. Sivanandam, S.N., and Deepa, S.N., "Introduction to Genetic Algorithms". Verlag Berlin Heidelberg, 2008.
8. "Guide to Long-Span Concrete Floors", Cement and Concrete Association of Australia, 2003.