Capacity survey of Nagarjuna Sagar reservoir, India using Linear Mixture Model (LMM) approach
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ABSTRACT
Satellite data has long been in use to estimate the water-spread area at different water levels of a reservoir. Traditional approaches such as maximum likelihood classification and band threshold method involve the per-pixel approach to delineate the water-spread area of a reservoir. One of the limitations of the per-pixel approach is that the pixels representing reservoir border, containing water with soil and vegetation, are also classified as water pixels, thereby giving inaccurate estimate of the water-spread area. To compute the water-spread area accurately, the sub-pixel or linear mixture model (LMM) approach has been adopted in this study. IRS-1C and 1D satellite image data (24m) of eight optimal dates ranging from minimum draw down level (MDDL) to full reservoir level (FRL) were used to estimate the water-spread area of the reservoir. The extracted water-spread areas using sub-pixel approach was in turn used to quantify the capacity of the Nagarjuna Sagar reservoir for the water year 2002. The estimated capacity of the reservoir using sub-pixel approach was 8014.49 Mm$^3$.

Key Words: Reservoir, water-spread area, capacity estimation, sub-pixel approach.

1. Introduction
Sediment deposition and its accumulation in a reservoir depend on the sediment load carried by rivers and reservoir storage capacity respectively (Phatarford,1990). Rivers in India carry only 5% of the global water runoff but they transport about 30% of the total sediment carried to the oceans (Milliman and Meade, 1983). Sediment transport of a number of Indian rivers has been reported by several authors (Abbas and Subramanian, 1984; Biksham and Subramanian, 1988; Chakrabani and Subramanian, 1990). The transported silt eventually gets deposited at different levels of a reservoir and reduces its storage capacity (Goel et al. 2002, Jain et al.2002, Sreenivasulu and Udayabaskar 2010). Reduction in the storage capacity beyond a limit prevents the reservoir from the fulfillment of the purpose for which it is designed. Periodical capacity surveys of the reservoir help in assessing the rate of sedimentation and reduction in storage capacity. The conventional technique such as hydrographic survey and inflow-outflow approaches, for the estimation of capacity of a reservoir are cumbersome, time consuming, expensive and involve more man power. An alternate to conventional methods, remote sensing technique provides cost and time effective estimation of the live capacity of a reservoir (Jain 2002). Multi-date satellite remote sensing data provide information on elevation contours, in the form of water-spread area, at different water levels of a reservoir. Water-spread area thus interpreted from the satellite data is used as an input in a simple volume estimation formula to calculate the capacity of a reservoir. Such works have been reported by Vibulsresth (1988) for Ubolratana Reservoir in Thailand, Manavalan (1990) for Malaprabha Reservoir in...
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For quantification of the capacity of a reservoir, the only thematic information that has to be extracted from the satellite data is the water-spread area at different water levels of the reservoir (Morris and Fan 1998, Peng 2006). The different approaches such as maximum likelihood, minimum distance to mean classification and band threshold method, to delineate various thematic information from the remote sensing digital data adopt the per-pixel based methodology and assign a pixel to a single land cover type (Jensen 1996, Bastin 1997) whereas in reality, a single pixel may contain more than one land cover (known as a mixed pixel). Mixed pixels are common especially near the boundaries of two or more discrete classes (Foody and Cox 1994, Oleson 1994). The boundary pixels of the water-spread area that are mixed in nature, representing soil, vegetation class with moisture are also classified as water pixels when a per-pixel based approach is applied, thereby producing inaccurate estimate of the water-spread area. To accurately compute the water-spread area to the maximum possible extent, thereby reducing the error in the estimation of capacity of a reservoir, a sub-pixel or linear mixture model (LMM) approach has been chosen for classifying the boundary pixels of water-spread area from different water levels of Nagarjuna Sagar reservoir located in Andhra Pradesh state of India.

2. Study reservoir

Nagarjuna Sagar is one of the world’s largest masonry dam built across the river Krishna in Nagarjuna Sagar town, Nalgonda District of Andhra Pradesh, India, between 1955 and 1967. The capacity of Nagarjuna Sagar reservoir at FRL (Full Reservoir Level) during impoundment was 11,472 million cubic metres. The dam is 490 ft (150 m) tall and 1.6 km long with 26 gates which are 42 ft (13 m) wide and 45 ft (14 m) tall. Nagarjuna Sagar was the earliest in the series of large infrastructure projects initiated for the Green Revolution in India; it is also one of the earliest multi-purpose projects in India. The dam provides irrigation water to the Nalgonda District, Prakasam District, Khammam District and Guntur District and generates electric power to the national grid. This region experiences hot and dry summer throughout the year except during the South-west monsoon season. The monsoon season is from June to September and retreating monsoon or the post monsoon season is during October to November. The average rainfall in the district is 772 mm. 71% of the annual rainfall is received during south-west monsoon. The mean daily maximum temperature is about 40º C and the mean daily minimum is about 28º C. The soils in and around the study reservoir mainly comprise loamy sands, sandy loams and sandy clay loams. In the areas of flat topography and alongside the river Krishna and its tributaries comprises mainly of black cotton soil. The district contains dry mixed deciduous forest. Southern Tropical thorn forest also most common type found in Nalgonda district. The major crops in the district are Paddy, Jowar, Bajra, Maize, Redgram, Greengram, Groundnut, Sesamum, Castor and Cotton.

2.1 Satellite data used

The image data used in this study were obtained by the Indian Remote Sensing (IRS) satellites IRS-1C & 1D (LISS-III sensor) which provides a spatial resolution of 24 m and spectral resolution in four different bands (0.52-0.59, 0.62-0.68, 0.77-0.86, 1.55-1.70 µm). The different dates of satellite data used and the respective water level during the pass of the satellite over the reservoir are given in Table 1. Reservoir water level data and the hydrographic survey details have been collected from the Nagarjuna sagar reservoir authority responsible for the maintenance and operation of the reservoir.
Table 1: Details of satellite data used and the water level during the pass of the satellite over the reservoir

<table>
<thead>
<tr>
<th>Sl.No.</th>
<th>Name of the Satellite</th>
<th>Date of Satellite Pass</th>
<th>Reservoir Elevation (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>IRS-1C</td>
<td>23.10.2001</td>
<td>175.32</td>
</tr>
<tr>
<td>2.</td>
<td>IRS-1D</td>
<td>12.12.2001</td>
<td>166.68</td>
</tr>
<tr>
<td>3.</td>
<td>IRS-1D</td>
<td>24.12.2001</td>
<td>164.74</td>
</tr>
<tr>
<td>4.</td>
<td>IRS-1D</td>
<td>25.02.2002</td>
<td>157.03</td>
</tr>
<tr>
<td>5.</td>
<td>IRS-1D</td>
<td>14.09.2002</td>
<td>156.94</td>
</tr>
<tr>
<td>6.</td>
<td>IRS-1D</td>
<td>11.05.2002</td>
<td>154.72</td>
</tr>
<tr>
<td>7.</td>
<td>IRS-1D</td>
<td>27.11.2002</td>
<td>153.19</td>
</tr>
<tr>
<td>8.</td>
<td>IRS-1D</td>
<td>22.12.2002</td>
<td>152.28</td>
</tr>
</tbody>
</table>

3. Methodology

The changes in water-spread could be accurately estimated by analyzing the areal spread of the reservoir at different elevations over a period of time using the satellite image data (Morris and Fan 1998, Smith and Pavelsky 2009). The sub-pixel or Linear Mixture Model (LMM) approach has been used in this study to extract the water-spread area of the reservoir. Estimated water-spread areas were used in a simple volume estimation formula to compute the storage capacity of the reservoir. Estimation of water-spread area and computation of capacity of the reservoir is discussed in the following sections.

3.1 Geo-referencing of satellite data

In the IRS-1C and 1D (LISS-III) satellite data the reservoir water-spread area was free from clouds and noise in all of the eight images used. The image scene of 23rd October 2001 was geo-referenced with respect to 1:50,000 survey of India topographic maps. The geo-referencing was done using ployconic projection and nearest neighborhood re-sampling technique to create a geo-referenced image of pixel size 24m x 24m. Subsequently the other satellite images corresponding to various water levels were also registered with the geo-referenced image using the image to image registration technique. In every image 20 to 25 ground control points were used, which resulted in a root mean squared error (RMSE) of 0.15 to 0.18 of a pixel.

3.2 Sub-pixel based approach

The sub-pixel classifier uses the linear unmixing technique, that allows in identifying the “material of interest” and determine its “material part fraction” or cover percentage, within a pixel.

Linear spectral unmixing is an excellent approximation for calculating the abundance or fraction of an end-member in an image pixel. This soft classification technique aims at estimating the proportions of specific classes that occur within each pixel using linear mixing approach (Bryant 1996, Foody 1996). In this study the reservoir water-spread area was estimated using linear spectral unmixing approach.

The basic assumption of linear mixture model is that the measured reflectance of a pixel is the linear sum of the reflectance of the components that make up the pixel. The basic hypothesis is also that the image spectra are the result of mixtures of surface materials, shade and clouds, and that each of these components is linearly independent of the other (Quarmby et al. 1992,
Borel et al. 1994). Linear unmixing also assumes that all the materials within the image have sufficient spectral contrast to allow their separation. In soft classification, the estimated variables (the fractions or proportions of each land cover class) are continuous, ranging from 0 to 100 percent coverage within a pixel. Settle and Drake (1993) and Foody and Cox (1994), proposed a mathematical expression for linear spectral unmixing. The theory behind this is the contribution of a series of end-members present within a pixel to its spectral signature. Hence, the spectral signature of a pixel would be derived from the sum of the products of the single spectrum of the end-members it contains, each weighted by a fraction plus a residue which would be explained by the following mathematical model:

\[
R_i = \sum f_k R_{ik} + E_i
\]  

(1)

where \( \sum f_k = 1 \)  

(2)

and \( 0 \leq f_k \leq 1 \)  

(3)

\( i = 1, \ldots, m \) (number of spectral bands)

\( k = 1, \ldots, n \) (number of endmembers)

\( R_i \) = Spectral reflectance of band i of a pixel which contains one or more endmembers

\( f_k \) = Proportion of endmember k within the pixel

\( R_{ik} \) = Known spectral reflectance of endmember k within the pixel on band i

\( E_i \) = Error for band i (Difference between the observed pixel reflectance \( R_i \) and the reflectance of that pixel computed from the model).

Equations 1 and 2 introduce the constraints that the sum of the fractions are equal to one and they are non-negative. To solve \( f_k \), the following conditions must be satisfied: (i) selected endmembers should be independent of each other, (ii) the number of endmembers should be less than or equal to the spectral bands used, and (iii) selected spectral bands should not be highly correlated.

In this study, the linear spectral unmixing is adopted based on the equations described below to segregate the actual information within a pixel of an image

\[
\begin{align*}
R_1 &= F_{\text{water}} \times R_{1\text{water}} + F_{\text{Veg}} \times R_{1\text{Veg}} + F_{\text{Soil}} \times R_{1\text{Soil}} + \epsilon_1 \\
R_2 &= F_{\text{water}} \times R_{2\text{water}} + F_{\text{Veg}} \times R_{2\text{Veg}} + F_{\text{Soil}} \times R_{2\text{Soil}} + \epsilon_2 \\
R_3 &= F_{\text{water}} \times R_{3\text{water}} + F_{\text{Veg}} \times R_{3\text{Veg}} + F_{\text{Soil}} \times R_{3\text{Soil}} + \epsilon_3
\end{align*}
\]

(4)

Where,

\( \rightarrow R_1, R_2 \) and \( R_3 \) represent the signal recorded at the satellite in the green, red and NIR bands of the LISS-III sensor.

\( \rightarrow F_{\text{water}}, F_{\text{Veg}} \) and \( F_{\text{Soil}} \) are the fraction of the pixel covered by water, vegetation, and soil.

\( \rightarrow R_{1\text{water}}, R_{2\text{water}} \) and \( R_{3\text{water}} \) represent the reflectance of water in each of the three spectral bands.

\( \rightarrow R_{1\text{Veg}}, R_{2\text{Veg}} \) and \( R_{3\text{Veg}} \) represent the reflectance of vegetation in each of the three
spectral bands.

- $R_{1\text{Soil}}, R_{2\text{Soil}}$ and $R_{3\text{Soil}}$ represent the reflectance of soil in each of the three spectral bands.
- $\epsilon_1, \epsilon_2$ and $\epsilon_3$ are the error components of band 1, 2, and 3.

The system of linear equations shown above can be solved by a least square solution which minimizes the sum of squares of errors.

The sub-pixel based approach was applied to find out the proportion or fraction of water class that exits in the periphery pixels of the reservoir. The first step executed in the sub-pixel approach was, selection of end-members. In general the border pixels may contain any combination and proportions of water, vegetation and soil classes, therefore these three classes were chosen to collect the end-members. Scatter plot method was used to identify the end-members. The identified end-member spectra were supplied as input to the linear mixture model (LMM) approach. The output of the model run contains three images known as water, soil and vegetation fraction images. Description of the fraction images is given in section 4.1.

3.3 Computation of volume between successive water levels

Traditionally the reservoir volume between two consecutive reservoir water levels, was computed using the prismoidal formula, the Simpson formula and the trapezoidal formulae (Patra 2001). Of these, the trapezoidal formula has been most widely used for computation of volume (Goel and Jain 2002, Rathore 2006). The water-spread area estimated using sub-pixel approach was used as an input in the volume estimation formula to find out the, volume at different water levels of the reservoir. In this study the volume between two consecutive reservoir water levels was computed using the following trapezoidal formula.

Trapezoidal Formula: $V = \frac{H}{3} (A_1 + A_2 + \sqrt{A_1A_2})$  \hspace{1cm} (5)

Where $V$ is the volume between two consecutive water levels. $A_1$ and $A_2$ are the water-spread area at the reservoir water level 1 and 2 respectively and $H$ is the difference between these two water levels.

3.4 Computation of storage capacity of the reservoir

The volume computed between different water levels (i.e from Minimum Draw Down Level to Full Reservoir Level) was added up to calculate the cumulative or storage capacity of the reservoir.

4. Results and discussion

4.1 Computation of reservoir capacity by sub-pixel approach

The fraction images (Figure 1) generated using the sub-pixel approach described in the methodology section contain a wealth of information about the reservoir. Each fraction image corresponds to a single land cover only. For example, the pixels in the water fraction image provide information only on the proportion or amount of water it contains. Likewise the vegetation and soil fraction image provide information on the proportions of the respective classes only. However, in this study the interest is only to know about, the amount of water present in the border pixels of the reservoir. The value of the pixels in the fraction image ranges...
from 0 to 1. A pixel from the water fraction image having a value of 0 indicates that there is no water at all in that pixel, whereas a pixel having a value of 0.28 indicates that 28% of the area of the pixel is occupied by water while a pixel value of 1 indicates that 100% of the area of the pixel is occupied by water (i.e the pixel is fully occupied by water). Therefore, for a pixel having a value of 0.65, the area of water occupied by that pixel is 374.4 m² (0.65 x 24m x 24m).

The pixels representing the reservoir border which have a minimum value of up to 0.1 in the water fraction image (i.e a pixel contains a minimum 10% of area of water) were isolated from the water fraction image and the area covered by water in these border pixels were estimated. The number of pixels that contains 100% of water was also found out. By summing up the area occupied by these two types of pixels, the total water-spread area, corresponding to a particular water level of the reservoir was computed. This exercise was carried out for all the eight images used in this study. The water-spread area thus estimated was again used as an input in the trapezoidal formula to compute the storage capacity or cumulative capacity of the Nagarjuna sagar reservoir using the sub-pixel classification approach.

Here it is worth mentioning that, a pixel containing 65% of water may be labelled as containing 100% of water by the per-pixel approach. In such case the water-spread area is over estimated. Conversely if the pixel contains 40% of water, then the entire pixel is not considered as a water pixel. Hence, the water-spread area is under estimated. Such errors due to over estimation or under estimation do not occur in the sub-pixel approach. Thus, the sub-pixel approach reduces the error imposed by the per-pixel approach. The estimated cumulative capacity of Nagarjuna sagar reservoir at the water level 175.32 m (Near FRL) using the sub-pixel approach was 8014.49 Mm³. The capacity estimated using sub-pixel approach is given in Table 2.

**Table 2: Capacity estimation of Nagarjuna sagar reservoir using the sub-pixel classification approach (2002)**

<table>
<thead>
<tr>
<th>Sl.No.</th>
<th>Date of Satellite Pass</th>
<th>Reservoir Elevation above m.s.l (m)</th>
<th>Water-spread area estimated using sub-pixel approach (Mm²)</th>
<th>Volume (Mm³)</th>
<th>Cumulative capacity (Mm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>23.10.2001</td>
<td>175.32</td>
<td>235.15</td>
<td>1864.59</td>
<td>8014.49</td>
</tr>
<tr>
<td>2.</td>
<td>12.12.2001</td>
<td>166.68</td>
<td>197.03</td>
<td>379.37</td>
<td>6149.90</td>
</tr>
<tr>
<td>3.</td>
<td>24.12.2001</td>
<td>164.74</td>
<td>194.08</td>
<td>1404.19</td>
<td>5770.53</td>
</tr>
<tr>
<td>4.</td>
<td>25.02.2002</td>
<td>157.03</td>
<td>170.43</td>
<td>15.20</td>
<td>4366.33</td>
</tr>
<tr>
<td>5.</td>
<td>14.09.2002</td>
<td>156.94</td>
<td>167.42</td>
<td>357.87</td>
<td>4351.13</td>
</tr>
<tr>
<td>6.</td>
<td>11.05.2002</td>
<td>154.72</td>
<td>155.06</td>
<td>232.61</td>
<td>3993.26</td>
</tr>
<tr>
<td>7.</td>
<td>27.11.2002</td>
<td>153.19</td>
<td>149.02</td>
<td>133.37</td>
<td>3760.66</td>
</tr>
<tr>
<td>8.</td>
<td>22.12.2002</td>
<td>152.28</td>
<td>144.11</td>
<td>3627.29</td>
<td></td>
</tr>
</tbody>
</table>
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5. Conclusions

High spatial-resolution image data enables accurate mapping of the terrain features. The use of high spatial resolution satellite image data, however, is constrained by factors such as cost and less area covered by the sensor. Hence, in hydrological applications estimating water-spread area may be difficult because a reservoir may not be imaged in a single pass of the satellite and atmospheric condition would be different from path to path (Hung 2005). An alternative method to overcome such constrains is the use of the sub-pixel based approach. The simplest methodology for such an approach is linear mixture modeling, which has been demonstrated in this study to estimate the capacity of Nagarjuna sagar reservoir in India.

Though sub-pixel approach found to be a better alternative to per-pixel approach, there are certain limitations such as the spatial location of the fractions within a pixel is unknown. In addition the sub-pixel classifier produces more accurate results only with hyperspectral images. Hence, the use of hyperspectral image data with higher spatial resolution would have yielded better results.

6. References


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